6. Integral

6.1 Indefinite Integral

Integration is an important concept in mathematics and, together with its inverse, differentiation, is one of the two main operations in calculus. Given a function *f* of a real variable *x* and an interval [*a*, *b*] of the real line, the **definite integral**

$$\int_a^b f(x)\,dx$$

is defined informally to be the signed area of the region in the *xy*-plane bounded by the graph of *f*, the *x*-axis, and the vertical lines *x* = *a* and *x* = *b*, such that area above the *x*-axis adds to the total, and that below the *x*-axis subtracts from the total.

The term *integral* may also refer to the related notion of the anti-derivative, a function *F* whose derivative is the given function *f*. In this case, it is called an *indefinite integral* and is written:

$$F(x) = \int f(x)\,dx.$$

However, the integrals discussed in this lesson are termed *definite integrals*.

The principles of integration were formulated independently by Isaac Newton and Gottfried Leibniz in the late 17th century. Through the fundamental theorem of calculus, which they independently developed, integration is connected with differentiation: if *f* is a continuous real-valued function defined on a closed interval [*a*, *b*], then, once an anti-derivative *F* of *f* is known, the definite integral of *f* over that interval is given by

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Integrals and derivatives became the basic tools of calculus, with numerous applications in science and engineering. The founders of the calculus thought of the integral as an infinite sum of rectangles of infinitesimal width. A rigorous mathematical definition of the integral was given by Bernhard Riemann. It is based on a limiting procedure which approximates the area of a curvilinear region by breaking the region into thin vertical slabs. Beginning in the nineteenth century, more sophisticated notions of integrals began to appear, where the type of the function as well as the domain over which the integration is performed has been generalized.

A line integral is defined for functions of two or three variables, and the interval of integration [*a*, *b*] is replaced by a certain curve connecting two points on the plane or in the space. In a surface integral, the curve is replaced by a piece of a surface in the three-dimensional space. Integrals of differential forms play a fundamental role in modern differential geometry.

These generalizations of integrals first arose from the needs of physics, and they play an important role in the formulation of many physical laws, notably those of electrodynamics. There are many modern concepts of integration, among these, the most common is based on the abstract mathematical theory known as Lebesgue integration, developed by Henri Lebesgue.

In calculus, an **antiderivative**, **primitive integral** or **indefinite integral** of a function *f* is a differentiable function *F* whose derivative is equal to *f*, i.e., $F' = f$. The process of solving for anti-derivatives is called **anti-differentiation** (or **indefinite integration**) and its opposite operation is called differentiation, which is the process of finding a derivative. Anti-derivatives are related to definite integrals through the fundamental theorem of calculus: the definite integral of a function over an interval is equal to the difference between the values of an anti-derivative evaluated at the endpoints of the interval.

6.2 **Definite integral**

The first documented systematic technique capable of determining integrals is the method of exhaustion of the ancient Greek astronomer Eudoxus (*ca.* 370 BC), which sought to find areas and volumes by breaking them up into an infinite number of shapes for which the area or volume was known.

This method was further developed and employed by Archimedes in the 3rd century BC and used to calculate areas for parabolas and an approximation to the area of a circle. Similar methods were independently developed in China around the 3rd century AD by Liu Hui, who used it to find the area of the circle. This method was later used in the 5th century by Chinese father-and-son mathematicians Zu Chongzhi and Zu Geng to find the volume of a sphere (Shea 2007; Katz 2004, pp. 125–126).

The next significant advances in integral calculus did not begin to appear until the 16th century. At this time the work of Cavalieri with his *method of indivisibles*, and work by Fermat, began to lay the foundations of modern calculus, with Cavalieri computing the integrals of x^n up to degree $n = 9$ in Cavalieri's quadrature formula. Further steps were made in the early 17th century by Barrow and Torricelli, who provided the first hints of a connection between integration and differentiation. Barrow provided the first proof of the fundamental theorem of calculus. Wallisgeneralized Cavalieri's method, computing integrals of x to a general power, including negative powers and fractional powers.

Although the Riemann and Lebesgue integrals are the most widely used definitions of the integral, a number of others exist, including:

- The Darboux integral which is equivalent to a Riemann integral, meaning that a function is Darboux-integrable if and only if it is Riemann-integrable, and the values of the two integrals, if they exist, are equal. Darboux integrals have the advantage of being simpler to define than Riemann integrals.
- The Riemann–Stieltjes integral, an extension of the Riemann integral.
- The Lebesgue–Stieltjes integral, further developed by Johann Radon, which generalizes the Riemann–Stieltjes and Lebesgue integrals.
- The Daniell integral, which subsumes the Lebesgue integral and Lebesgue–Stieltjes integral without the dependence on measures.
- The Haar integral, used for integration on locally compact topological groups, introduced by Alfréd Haar in 1933.
- The Henstock–Kurzweil integral, variously defined by Arnaud Denjoy, Oskar Perron, and (most elegantly, as the gauge integral) Jaroslav Kurzweil, and developed by Ralph Henstock.
- The Itō integral and Stratonovich integral, which define integration with respect to semimartingales such as Brownian motion.
- The Young integral, which is a kind of Riemann–Stieltjes integral with respect to certain functions of unbounded variation.
- The rough path integral defined for functions equipped with some additional "rough path" structure, generalizing stochastic integration against both semimartingales and processes such as the fractional Brownian motion

In addition we also have the definite integral.

The development of the definition of the definite integral begins with a function $f(x)$, which is continuous on a closed interval $[a, b]$. The given interval is partitioned into " n" subintervals that, although not necessary, can be taken to be of equal lengths (Δx). An arbitrary domain value, x_i, is chosen in each subinterval, and its subsequent function value, $f(x_i)$, is determined. The product of each function value times the corresponding subinterval length is determined, and these " n" products are added to determine their sum. This sum is referred to as a **Riemann sum** and may be positive, negative, or zero, depending upon the behavior of the function on the closed interval. For example, if $f(x) > 0$ on $[a, b]$, then the Riemann sum will be a positive real number. If $f(x) < 0$ on $[a, b]$, then the Riemann sum will be a negative real number

6.3 **Research Methods**

The desirability of integrating quantitative and qualitative research methods in development work is widely acknowledged, but the successful implementation of integrated approaches in the field has often proved elusive. However, there is now a growing body of experience in the development field demonstrating the benefits which can be achieved from multi-method research integrating quantitative and qualitative methods, some of which is reported in this volume.

However, despite significant progress in promoting integrated approaches, many researchers from both quantitative and qualitative traditions still often find it difficult to make full use of the data collection methods and analysis from the other tradition.

Some *quantitative survey researchers* may find it difficult to make full use of the wealth of case studies, PRA maps, calendars, and key informant interviews they have commissioned. On the other hand, ***qualitative researchers*** often complain that their findings, may be dismissed by survey researchers as not being sufficiently representative or rigorous. Qualitative researchers also express the concern that even after collaborative research efforts, the survey researchers still do not understand the true nature of a complex phenomenon such as poverty.

The message is that there is a growing consensus on the value of integrated approaches, but that further work needed to develop guidelines for the effective use of these integrated approaches.

6.4 **Relationship of socioeconomic comprehensive administrative sciences**

Administrative science is the study of the management of business and other organizations. It refers to two meanings: first, it is concerned with the implementation of policy; second, it is an academic discipline that studies this implementation and prepares civil servants for working in the public service. As a "field of inquiry with a diverse scope" its "fundamental goal... is to advance management and policies so that organizations can function." Some of the various definitions which have been offered for the term are: "the management of organizational programs"; the "translation of policies into the reality that citizens see every day"; and "the study of decision making, the analysis of the policies themselves, the various inputs that have produced them, and the inputs necessary to produce alternative policies.

Socioeconomics (also known as **socio-economics** or **social economics**) is the social science that studies how economic activity affects social processes. In general it analyzes how societies progress, stagnate, or regress because of their local or regional economy, or the global economy.

With that definition we see the relationship between socioeconomics and administrative sciences in that one is management through policy with people and the other is the economics concerning people.

Socioeconomics is sometimes used as an umbrella term with different usages. The term 'Social economics' may refer broadly to the "use of economics in the study of society."[1] More narrowly, contemporary practice considers behavioral interactions of individuals and groups through social capital and social "markets" (not excluding for example, sorting by marriage) and the formation of social norms.[2] In the latter, it studies the relation of economics to social values.[3]

A distinct supplemental usage describes social economics as "a discipline studying the reciprocal relationship between economic science on the one hand and social philosophy, ethics, and human dignity on the other" toward social reconstruction and improvement[4] or as also emphasizing multidisciplinary methods from such fields as sociology, history, and political science.

In many cases, socio-economists focus on the social impact of some sort of economic change. Such changes might include a closing factory, market manipulation, the signing of international trade treaties, new natural gas regulation, etc. Such social effects can be wide-ranging in size, anywhere from local effects on a small community to changes to an entire society.

Examples of causes of socioeconomic impacts include new technologies such as cars or mobile phones, changes in laws, changes in the physical environment (such as increasing crowding within cities), and ecological changes (such as prolonged drought or declining fish stocks). These may affect patterns of consumption, the distribution of incomes and wealth, the way in which people behave (both in terms of purchase decisions and the way in which they choose to spend their time), and the overall quality of life.