9.1 What Is Statistical Quality Control?
In Chapter 5 we learned that total quality management (TQM) addresses organizational quality from managerial and philosophical viewpoints. TQM focuses on customer driven quality standards, managerial leadership, continuous improvement, quality built into product and process design, identifying quality problems at the source, and making quality everyone’s responsibility. However, talking about solving quality problems is not enough. We need specific tools that can help us make the right quality decisions. These tools come from the field of statistics and are used to help identify quality problems in the production process as well as in the product itself. Statistical quality control is the subject of this chapter.

Statistical quality control (SQC) is the term used to describe the set of statistical tools used by quality professionals. Statistical quality control can be divided into three broad categories:

1. **Descriptive statistics** are used to describe quality characteristics and relationships. Included are statistics such as the mean, the standard deviation, the range, and a measure of the distribution of data.

2. **Statistical process control** (SPC) involves inspecting a random sample of the output from a process and deciding whether the process is producing products with characteristics that fall within a predetermined range. SPC answers the question of whether or not the process is functioning properly.

3. **Acceptance sampling** is the process of randomly inspecting a sample of goods and deciding whether to accept the entire lot based on the results. Acceptance sampling determines whether a batch of goods should be accepted or rejected. The tools in each of these categories provide different types of information for use in analyzing quality. Descriptive statistics are
used to describe certain quality characteristics, such as the central tendency and variability of observed data.

Although descriptions of certain characteristics are helpful, they are not enough to help us evaluate whether there is a problem with quality. Acceptance sampling can help us do this. It helps us decide whether desirable quality has been achieved for a batch of products and whether to accept or reject the items produced. Although this information is helpful in making the quality acceptance decision after the product has been produced, it does not help us identify and catch a quality problem during the production process. For this we need tools in the statistical process control (SPC) category.

All three of these statistical quality control categories are helpful in measuring and evaluating the quality of products or services. However, statistical process control (SPC) tools are used most frequently because they identify quality problems during the production process. For this reason, we will devote most of the chapter to this category of tools. The quality control tools we will be learning about do not only measure the value of a quality characteristic; they also help us identify a change or variation in some quality characteristic of the product or process. We will first see what types of variation we can observe when measuring quality. Then we will be able to identify the specific tools to use for measuring this variation.

Variation in the production process leads to quality defects and lack of product consistency. The Intel Corporation, the world’s largest and most profitable manufacturer of microprocessors, understands this. Therefore, Intel has implemented a program it calls “copy-exactly” at all its manufacturing facilities. The idea is that regardless of whether the chips are made in Arizona, New Mexico, Ireland, or any of its other plants, they are made in exactly the same way. This means using the same equipment, the same exact materials, and performing the same tasks in the exact same order.

The level of detail to which the “copy exactly” concept goes is meticulous. For example, when a chip making machine was found to be a few feet longer at one facility than another, Intel made them match. When water quality was found to be different at one facility, Intel instituted a purification system to eliminate any differences. Even when a worker was found polishing equipment in one direction, he was asked to do it in the approved circular pattern. Why such attention to exactness of detail? The reason is to minimize all variation. Now let’s look at the different types of variation that exist.
9.2 **Sources of Variation: Common and Assignable Causes**

A statistic that measures the central tendency of a set of data. If you look at bottles of a soft drink in a grocery store, you will notice that no two bottles are filled to exactly the same level. Some are filled slightly higher and some slightly lower. Similarly, if you look at blueberry muffins in a bakery, you will notice that some are slightly larger than others and some have more blueberries than others. These types of differences are completely normal. No two products are exactly alike because of slight differences in materials, workers, machines, tools, and other factors. These are called common, or random, **causes of variation**. Common causes of variation are based on random causes that we cannot identify. These types of variation are unavoidable and are due to slight differences in processing.

An important task in quality control is to find out the range of natural random variation in a process. For example, if the average bottle of a soft drink called Cocoa Fizz contains 16 ounces of liquid, we may determine that the amount of natural variation is between 15.8 and 16.2 ounces. If this were the case, we would monitor the production process to make sure that the amount stays within this range. If production goes out of this range—bottles are found to contain on average 15.6 ounces—this would lead us to believe that there is a problem with the process because the variation is greater than the natural random variation.

The second type of variation that can be observed involves those where the causes can be precisely identified and eliminated. These are called **assignable causes of variation**. Examples of this type of variation are poor quality in raw materials, an employee who needs more training, or a machine in need of repair. In each of these examples, the problem can be identified and corrected. If the variation is allowed to persist, it will continue to create a problem in the quality of the product. In the example of the soft-drink bottling operation, bottles filled with 15.6 ounces of liquid would signal a problem. The machine may need to be readjusted, an assignable cause of variation. We can assign the variation to a particular cause (machine needs to be readjusted) and we can correct the problem (readjust the machine).

9.3 **Descriptive Statistics**

Descriptive statistics can be helpful in describing certain characteristics of a product and a process. The most important descriptive statistics are measures of central tendency such as the mean, measures of variability such as the standard deviation and range, and measures of the distribution of data. We first review these
9.4 **The Mean**
In the soft-drink bottling example, we stated that the average bottle is filled with 16 ounces of liquid. The arithmetic average, or the **mean**, is a statistic that measures the central tendency of a set of data. Knowing the central point of a set of data is highly important. Just think how important that number is when you receive test scores! To compute the mean, we simply sum all the observations and divide by the total number of observations.

9.4.1 **The Range and Standard Deviation**
In the bottling example, we also stated that the amount of natural variation in the bottling process is between 15.8 and 16.2 ounces. This information provides us with the amount of variability of the data. It tells us how spread out the data are around the mean. There are two measures that can be used to determine the amount of variation in the data. The first measure is the **range**, which is the difference between the largest and smallest observations. In our example, the range for natural variation is 0.4 ounces. Another measure of variation is the **standard deviation**.

9.4.2 **Distribution of Data**
A third descriptive statistic used to measure quality characteristics is the shape of the distribution of the observed data. When a distribution is symmetric, there are the same number of observations below and above the mean. This is what we commonly find when only normal variation is present in the data. When a disproportionate number of observations are either above or below the mean, we say that the data have a **skewed distribution**.

9.5 **Statistical Process Control Methods**
Statistical process control methods employ descriptive statistics to monitor the quality of the product and process. As we have learned so far, there are common and assignable causes of variation in the production of every product. Using statistical process control, we want to determine the amount of variation that is common or normal. Then we monitor the production process to make sure production stays within this normal range. That is, we want to make sure the process is in a **state of control**. The most commonly used tool for monitoring the production process is a control chart. Different types of control charts are used to monitor different aspects of the production process. In this section we will learn how to develop and use control charts.
9.5.1 Developing Control Charts
A control chart (also called process chart or quality control chart) is a graph that shows whether a sample of data falls within the common or normal range of variation. A control chart has upper and lower control limits that separate common from assignable causes of variation. The common range of variation is defined by the use of control chart limits. We say that a process is out of control when a plot of data reveals that one or more samples fall outside the control limits.

Similarly, the lower control limit (LCL) is the minimum acceptable variation from the mean for a process that is in a state of control. In our example, the upper and lower control limits are 16.2 and 15.8 ounces, respectively. You can see that if a sample of observations falls outside the control limits, we need to look for assignable causes.

9.5.2 Control Charts for Variables
Control charts for variables monitor characteristics that can be measured and have a continuous scale, such as height, weight, volume, or width. When an item is inspected, the variable being monitored is measured and recorded. For example, if we were producing candles, height might be an important variable, so we could take samples of candles and measure their heights. Two of the most commonly used control charts for variables monitor both the central tendency of the data (the mean) and the variability of the data (either the standard deviation or the range). Note that each chart monitors a different type of information. When observed values go outside the control limits, the process is assumed not to be in control. Production is stopped, and employees attempt to identify the cause of the problem and correct it. Next we look at how these charts are developed.

9.5.3 Mean (x-Bar) Charts
A mean control chart is often referred to as an x-bar chart. It is used to monitor changes in the mean of a process. To construct a mean chart, we first need to construct the center line of the chart. To do this we take multiple samples and compute their means. Usually these samples are small, with about four or five observations.

9.5.4 Range (R) Charts
Range (R) Charts are another type of control chart for variables. Whereas x-bar charts measure a shift in the central tendency of the process, range charts monitor the dispersion or variability of the process. The method for developing and using R-charts is the same as that for x-bar charts.
9.5.5 Using Mean and Range Charts Together
You can see that mean and range charts are used to monitor different variables. The mean or $x$-bar chart measures the central tendency of the process, whereas the range chart measures the dispersion or variance of the process. Since both variables are important, it makes sense to monitor a process using both mean and range charts. It is possible to have a shift in the mean of the product but not a change in the dispersion.

For example, at the Cocoa Fizz bottling plant the machine setting can shift so that the average bottle filled contains not 16.0 ounces but 15.9 ounces of liquid. The dispersion could be the same, and this shift would be detected by an $x$-bar chart but not by a range chart.

9.5.6 Control Charts for Attributes
Control charts for attributes are used for quality characteristics that are counted rather than measured. Attributes are discrete in nature and entail simple yes-or-no decisions, for example, the number of nonfunctioning light bulbs, the proportion of broken eggs in a carton, the number of rotten apples, the number of scratches on a tile, or the number of complaints received. Two of the most common types of control charts for attributes are $p$-charts and $c$-charts.

$P$-charts are used to measure the proportion of items in a sample that are defective. Examples are the proportion of broken cookies in a batch and the proportion of cars produced with a misaligned fender. $P$-charts are appropriate when both the number of defectives measured and the size of the total sample can be counted. A proportion can then be computed and used as the statistic of measurement.

$C$-charts count the actual number of defects. For example, we can count the number of complaints from customers in a month, the number of bacteria in a petri dish, or the number of barnacles on the bottom of a boat. However, we cannot compute the proportion of complaints from customers, the proportion of bacteria in a petri dish, or the proportion of barnacles on the bottom of a boat. To summarize:

$P$-charts: Used when observations are placed in either of two groups. Examples:

- Defective or not defective
- Good or bad
- Broken or not broken

$C$-charts: Used when defects can be counted per unit of measure.
Examples:

- Number of dents per item
- **Number of complaints** per unit of time (e.g., hour, month, year)
- **Number of tears** per unit of area (e.g., square foot, square meter)

*Problem-Solving Tip* The primary difference between using a *p*-chart and a *c*-chart is as follows.

A *p*-chart is used when both the total sample size and the number of defects can be computed. A *c*-chart is used when we can compute only the number of defects but cannot compute the proportion that is defective.

**P-charts** are used to measure the proportion that is defective in a sample. The computation of the center line as well as the upper and lower control limits is similar to the computation for the other kinds of control charts. The center line is computed as the average proportion defective in the population. This is obtained by taking a number of sample observations at random and computing the average value of *p* across all samples. To construct the upper and lower control limits for a *p*-chart, we use the following formulas:

**C-charts** are used to monitor the number of defects per unit. Examples are the number of returned meals in a restaurant, the number of trucks that exceed their weight limit in a month, the number of discolorations on a square foot of carpet, and the number of bacteria in a milliliter of water. Note that the types of units of measurement we are considering are a period of time, a surface area, or a volume of liquid. The average number of defects, is the center line of the control chart. The upper and lower control limits are computed as follows:

### 9.6 Process Capability
So far, we have discussed ways of monitoring the production process to ensure that it is in a *state of control* and that there are no assignable causes of variation. A critical aspect of statistical quality control is evaluating the ability of a production process to meet or exceed preset specifications. This is called **process capability**. To understand exactly what this means, let’s look more closely at the term *specification*. **Product specifications**, often called *tolerances*, are preset ranges of acceptable quality characteristics, such as product dimensions. For a product to be considered acceptable, its characteristics must fall within this preset range. Otherwise, the product is not acceptable.

Specifications for a product are preset on the basis of how the product is going to be used or what customer expectations are. As we have learned, any production process has a certain amount of natural variation associated with it. To be capable of producing an acceptable product, the process variation cannot exceed the preset
specifications. Process capability thus involves evaluating process variability relative to preset product specifications in order to determine whether the process is capable of producing an acceptable product. In this section we will learn how to measure process capability.

6.7 Six Sigma Quality
The term Six Sigma was coined by the Motorola Corporation in the 1980s to describe the high level of quality the company was striving to achieve. Sigma (σ) stands for the number of standard deviations of the process. To achieve the goal of Six Sigma, Motorola has instituted a quality focus in every aspect of its organization. Before a product is designed, marketing ensures that product characteristics are exactly what customers want.

Operations ensures that exact product characteristics can be achieved through product design, the manufacturing process, and the materials used. The Six Sigma concept is an integral part of other functions as well. It is used in the finance and accounting departments to reduce costing errors and the time required to close the books at the end of the month. Numerous other companies, such as General Electric, Lockheed Martin, Allied Signal, American Express, and Texas Instruments, have followed Motorola’s leadership and have also instituted the Six Sigma concept.

In fact, the Six Sigma quality standard has become a benchmark in many industries. There are two aspects to implementing the Six Sigma concept.

- The first is the use of technical tools to identify and eliminate causes of quality problems. In fact, Six Sigma relies heavily on quantitative and data-driven technical tools.
- The second aspect of Six Sigma implementation is people involvement. In Six Sigma, all employees have the training to use technical tools and are responsible for rooting out quality problems. Employees are given martial arts titles that reflect their skills in the Six Sigma process.

Black belts and master black belts are individuals who have extensive training in the use of technical tools and are responsible for carrying out the implementation of Six Sigma. They are experienced individuals who oversee the measuring, analyzing, process controlling, and improving. They achieve this by acting as coaches, team leaders, and facilitators of the process of continuous improvement.
Green belts are individuals who have sufficient training in technical tools to serve on teams or on small, individual projects. The Six Sigma approach is organized around a five-step plan known as DMAIC, which stands for Define, Measure, Analyze, Improve, and Control:

Step 1: Define the quality problem of the process.
Step 2: Measure the current performance of the process.
Step 3: Analyze the process to identify the root cause of the quality problem.
Step 4: Improve the process by eliminating the root causes of the problem.
Step 5: Control the process to ensure the improvements continue.

The first three steps provide a study of the existing process, whereas the last two steps are involved in process change. All steps extensively utilized quantitative tools, such as measuring the current performance and analyzing the process for root causes of problems. You can also see that like the PDSA cycle described in Chapter 5, this is a circular process that is ongoing and never ends. Part of Six Sigma is to continuously search for quality problems and improve upon them. In organizations, this effort is led by the black belts.

Successful Six Sigma implementation requires commitment from top company leaders. These individuals must promote the process, eliminate barriers to implementation, and ensure that proper resources are available. A key individual is a champion of Six Sigma. This is a person who comes from the top ranks of the organization and is responsible for providing direction and overseeing all aspects of the process.