Economics for Engineers Future Present Worth Method

5.1 <u>Introduction</u>

In the future worth method of comparison of alternatives, the future worth of various alternatives will be computed. Then, the alternative with the maximum future worth of net revenue or with the minimum future worth of net cost will be selected as the best alternative for implementation. Decisions made during the engineering design phase of product development determine the majority (some say 85%) of the costs of manufacturing that product. Thus, a competent engineer in the 21st century must have an understanding of the principles of economics as well as engineering. This chapter examines the most important economic concepts that should be understood by engineers.

Engineers participate in a variety of decision-making processes, ranging from manufacturing to marketing to financing decisions. They must make decisions involving materials, plant facilities, the in house capabilities of company personnel, and the effective use of capital assets such as buildings and machinery. One of the engineer's primary tasks is to plan for the acquisition of equipment (fixed asset) that will enable the firm to design and produce products economically. These decisions are called *engineering economic decisions*.

5.2 <u>Revenue-Dominated Cash Flow</u>

A typical engineering economic decision involves two dissimilar types of dollar amounts. First, there is the investment, which is usually made in a lump sum at the beginning of the project, a time that for analytical purposes is called today, or time 0. Second, there is a stream of cash benefits that are expected to result from this investment over a period of future years.

In such a fixed asset investment funds are committed today in the expectation of earning a return in the future. In the case of a bank loan, the future return takes the form of interest plus repayment of the principal. This is known as the *loan cash flow*. In the case of the fixed asset, the future return takes the form of cash generated by productive use of the asset. The representation of these future

earnings along with the capital expenditures and annual expenses (such as wages, raw materials, operating costs, maintenance costs, and income taxes) is the *project cash flow*. This similarity between the loan cash flow and the project cash flow brings us an important conclusion—that is, first we need to find a way to evaluate a money series occurring at different points in time. Second, if we understand how to evaluate a loan cash flow series, we can use the same concept to evaluate the project cash flow series.

A generalized revenue-dominated cash flow diagram to demonstrate the future worth method of comparison is presented in Fig. 5.1.



In Fig. 5.1, P represents an initial investment, Rj the net-revenue at the end of the jth year, and S the salvage value at the end of the nth year. The formula for the future worth of the above cash flow diagram for a given interest rate, i is

$$FW(i) = -P(1+i)^{n} + R1(1+i)^{n-1} + R2(1+i)^{n-2} + \dots + Rj(1+i)^{n-j} + \dots + Rn + S$$

In the above formula, the expenditure is assigned with negative sign and the revenues are assigned with positive sign. If we have some more alternatives which are to be compared with this alternative, then the corresponding future worth amounts are to be computed and compared. Finally, the alternative with the maximum future worth amount should be selected as the best alternative.

5.3 Cost-Dominated Cash Flow

Money left in a savings account earns interest so that the balance over time is greater than the sum of the deposits. In the financial world, money itself is a commodity, and like other goods that are bought and sold, money costs money. The cost of money is established and measured by an *interest rate*, a percentage

that is periodically applied and added to an amount (or varying amounts) of money over a specified length of time. When money is borrowed, the interest paid is the charge to the borrower for the use of the lender's property; when money is loaned or invested, the interest earned is the lender's gain from providing a good to another. *Interest*, then, may be defined as the cost of having money available for use.

The operation of interest reflects the fact that money has a time value. This is why amounts of interest depend on lengths of time; interest rates, for example, are typically given in terms of a percentage per year. This principle of the time value of money can be formally defined as follows: the economic value of a sum depends on when it is received. Because money has earning power over time (it can be put to work, earning more money for its owner), a dollar received today has a greater value than a dollar received at some future time.

The changes in the value of a sum of money over time can become extremely significant when we deal with large amounts of money, long periods of time, or high interest rates. For example, at a current annual interest rate of 10%, \$1 million will earn \$100,000 in interest in a year; thus, waiting a year to receive \$1 million clearly involves a significant sacrifice. In deciding among alternative proposals, we must take into account the operation of interest and the time value of money to make valid comparisons of different amounts at various times.

A generalized cost-dominated cash flow diagram to demonstrate the future worth method of comparison is given in Fig. 5.2.



Fig. 5.2 Cost-dominated cash flow diagram.

In Fig. 5.2, P represents an initial investment, Cj the net cost of operation and maintenance at the end of the jth year, and S the salvage value at the end of the nth year. The formula for the future worth of the above cash flow diagram for a given interest rate, i is

$$FW(i) = P(1+i)^n + C1(1+i)^{n-1} + C2(1+i)^{n-2} + \dots + Cj(1+i)^{n-j} + \dots + Cn - S$$

In this formula, the expenditures are assigned with positive sign and revenues with negative sign. If we have some more alternatives which are to be compared with this alternative, then the corresponding future worth amounts are to be computed and compared. Finally, the alternative with the minimum future worth amount should be selected as the best alternative.

5.4 <u>The Elements of Transactions Involving Interest</u>

Many types of transactions involve interest — for example, borrowing or investing money, purchasing machinery on credit — but certain elements are common to all of them:

- 1. Some initial amount of money, called the *principal* (*P*) in transactions of debt or investment
- 2. The *interest rate* (*i*), which measures the cost or price of money, expressed as a percentage per period of time
- 3.A period of time, called the *interest period* (or *compounding period*), that determines how frequently interest is calculated
- 4. The specified length of time that marks the duration of the transaction and thereby establishes acertain *number of interest periods* (*N*)
- 5. A *plan for receipts or disbursements* (A_n) that yields a particular cash flow pattern over the length of time (for example, we might have a series of equal monthly payments [A] that repay a loan)
- 6. A *future amount of money* (F) that results from the cumulative effects of the interest rate over a number of interest periods

5.4.1 Cash Flow Diagrams

It is convenient to represent problems involving the time value of money in graphic form with a cash flow diagram (see Figure 17.2.1), which represents time by a horizontal line marked off with the number of interest periods specified. The cash flows over time are represented by arrows at the relevant periods: upward arrows for positive flows (receipts) and downward arrows for negative flows (disbursements).



FIGURE 17.2.1 A cash flow diagram for a loan transaction — borrow \$20,000 now and pay off the loan with five equal annual installments of \$5,141.85. After paying \$200 for the loan origination fee, the net amount of financing is \$19,800. The borrowing interest rate is 9%.

5.4.2 End-of-Period Convention

In practice, cash flows can occur at the beginning or in the middle of an interest period, or at practically any point in time. One of the simplifying assumptions we make in engineering economic analysis is the *end-of-period convention*, which is the practice of placing all cash flow transactions at the end of an interest period. This assumption relieves us of the responsibility of dealing with the effects of interest within an interest period, which would greatly complicate our calculations.

5.4.3 Compound Interest

Under the compound interest scheme, the interest in each period is based on the total amount owed at the end of the previous period. This total amount includes the original principal plus the accumulated interest that has been left in the account. In this case, you are in effect increasing the deposit amount by the amount of interest earned. In general, if you deposited (invested) P dollars at interest rate i, you would have P + iP = P(1 + i) dollars at the end of one period. With the entire amount (principal and interest) reinvested at the same rate i for another period, you would have, at the end of the second period,

$$P(1+i) + i P[(1+i)] = P(1+i)(1+i)$$
$$= P(1+i)^{2}$$

This interest-earning process repeats, and after N periods, the total accumulated value (balance) F will grow to

$$F = P(1+i)^N$$
(17.2.1)

5.5 Equivalence Calculations

Economic equivalence refers to the fact that a cash flow — whether it is a single payment or a series of payments — can be said to be converted to an *equivalent* cash flow at any point in time; thus, for any sequence of cash flows, we can find an equivalent single cash flow at a given interest rate and a given time.

Equivalence calculations can be viewed as an application of the compound interest relationships developed in Equation 17.2.1. The formula developed for calculating compound interest, $F = P(1 + i)^N$, expresses the equivalence between some present amount, P, and a future amount, F, for a given interest rate, i, and a number of interest periods, N. Therefore, at the end of a 3-year investment period at 8%, \$1000 will grow to

$$(1000 1(+0.08)^3 = 1259.71)$$

Thus at 8% interest, \$1000 received now is equivalent to \$1,2519.71 received in 3 years and we could trade \$1000 now for the promise of receiving \$1259.71 in 3 years. Example 17.2.1 demonstrates the application of this basic technique.

Example 17.2.1 — Equivalence

Suppose you are offered the alternative of receiving either \$3000 at the end of 5 years or P dollars today. There is no question that the \$3000 will be paid in full (no risk). Having no current need for the money, you would deposit the P dollars in an account that pays 8% interest. What value of P would make you indifferent in your choice between P dollars today and the promise of \$3000 at the end of 5 years from now?

Discussion. Our job is to determine the present amount that is economically equivalent to \$3000 in 5 years, given the investment potential of 8% per year. Note that the problem statement assumes that you would exercise your option of using the earning power of your money by depositing it. The "indifference" ascribed to you refers to economic indifference; that is, within a marketplace

where 8% is the applicable interest rate, you could trade one cash flow for the other.

Solution. From Equation (17.2.1), we establish

$$3000 = P(1+0.08)^5$$

Rearranging to solve for P,

 $P = \$3000 \ 1(+0.08)^5 = \2042

Comments. In this example, it is clear that if P is anything less than \$2042, you would prefer the promise of \$3000 in 5 years to P dollars today; if P were greater than \$2042, you would prefer P. It is less obvious that at a lower interest rate, P must be higher to be equivalent to the future amount. For example, at i = 4%, P = \$2466.

In this section, several examples highlighting the applications of the future worth method of comparison are presented.

EXAMPLE 5.1Consider the following two mutually exclusive alternatives:

	End of year				
Alternative	0	1	2	3	4
A (Rs.)	-50,00,000	20,00,000	20,00,000	20,00,000	20,00,000
B (Rs.)	-45,00,000	18,00,000	18,00,000	18,00,000	18,00,000

At i = 18%, select the best alternative based on future worth method of comparison.

SolutionAlternative A

Initial investment, P = Rs. 50,00,000

Annual equivalent revenue, A = Rs. 20,00,000

Interest rate, *i* = 18%, compounded annually

Life of alternative A = 4 years

The cash flow diagram of alternative A is shown in Fig. 5.3.



The future worth amount of alternative B is computed as $FW_A(18\%) = -50,00,000(F/P, 18\%, 4) + 20,00,000(F/A, 18\%, 4)$ = -50,00,000(1.939) + 20,00,000(5.215)= Rs. 7,35,000

Alternative B

Initial investment, P = Rs. 45,00,000

Annual equivalent revenue, A = Rs. 18,00,000

Interest rate, i = 18%, compounded annually Life of

alternative B = 4 years

The cash flow diagram of alternative B is illustrated in Fig. 5.4.



The future worth amount of alternative B is computed as $FW_B(18\%) = -45,00,000(F/P, 18\%, 4) + 18,00,000(F/A, 18\%, 4)$ = -45,00,000(1.939) + 18,00,000(5.215)= Rs. 6,61,500

The future worth of alternative A is greater than that of alternative B. Thus, alternative A should be selected.

EXAMPLE 5.2 A man owns a corner plot. He must decide which of the several alternatives to select in trying to obtain a desirable return on his investment. After much study and calculation, he decides that the two best alternatives are as given in the following table:

0	0		
		Build	Build soft
		gas station	ice-cream

		stand
First cost (Rs.)	20,00,000	36,00,000
Annual property taxes (Rs.)	80,000	1,50,000
Annual income (Rs.)	8,00,000	9,80,000
Life of building (years)	20	20
Salvage value (Rs.)	0	0

Evaluate the alternatives based on the future worth method at i = 12%.

Alternative 1—Build gas station

First cost = Rs. 20,00,000

Net annual income = Annual income – Annual property tax = Rs. 8,00,000 – Rs. 80,000

= Rs. 7,20,000

Life = 20 years

Interest rate = 12%, compounded annually The cash flow diagram for this alternative is depicted in Fig. 5.5.



The future worth of alternative 1 is computed as $FW_1(12\%) = -20,00,000(F/P, 12\%, 20) + 7,20,000(F/A, 12\%, 20)$ = -20,00,000(9.646) + 7,20,000 (72.052) = Rs. 3,25,85,440Alternative 2—Build soft ice-cream stand First cost = Rs. 36,00,000 Net annual income = Annual income – Annual property tax = Rs. 9,80,000 - Rs. 1,50,000 = Rs. 8,30,000 Life = 20 years Interest rate = 12%, compounded annually The cash flow diagram for this alternative is shown in Fig. 5.6.



Fig. 5.6 Cash flow diagram for alternative 2.

The future worth of alternative 2 is calculated as $FW_2(12\%) = -36,00,000(F/P, 12\%, 20) + 8,30,000(F/A, 12\%, 20)$ = -36,00,000(9.646) + 8,30,000(72.052)= Rs. 2,50,77,560

The future worth of alternative 1 is greater than that of alternative 2. Thus, building the gas station is the best alternative.

EXAMPLE 5.3 The cash flow diagram of two mutually exclusive alternatives are given in Figs. 5.7 and 5.8.



(a) Select the best alternative based on future worth method at i = 8%. (b) Rework part (a) with i = 9% and 20% (a) Evaluation at i = 8%Alternative 1—This comes under equal payment gradient series. *P* = Rs. 5,00,000 *A*1 = Rs. 50,000 *G* = Rs. 50,000 *i* = 8% n = 6 years The formula for the future worth of alternative 1 is $FW_1(8\%) = -P(F/P, 8\%, 6) + [A1 + G(A/G, 8\%, 6)]$ (F/A, 8%, 6) = -5,00,000(1.587) + [50,000 + 50,000(2.2764)] 7.336 = -79,35,000 + 1,63,820 7.336 = -79,35,000 + 12,01,784 = Rs. 4,08,283.52 Alternative 2—This comes under equal payment gradient series. *P* = Rs. 7,00,000 *A*1 = Rs. 70,000 *G* = Rs. 70,000 *i* = 8% n = 6 years The formula for the future worth of alternative 2 is $FW_2(8\%) = -P(F/P, 8\%, 6) + [A1 + G(A/G, 8\%, 6)] (F/A, 8\%, 6) FW_2(8\%) = -7,00,000$ 1.587 + [70,000 + 70,000 2.2764] 7.336 = -11, 10, 900 + 16, 82, 497= Rs. 5,71,596.93 The future worth of alternative 2 is more than that of alternative 1. Therefore, alternative 2 must be selected. (b) (i) Evaluation at i = 9%: Alternative 1 *P* = Rs. 5,00,000 A1 = Rs. 50,000 $G = \text{Rs.} 50,000 \ n = 6 \text{ years}$ The formula for the future worth of alternative 1 is as follows: $FW_1(9\%) = -P(F/P, 9\%, 6) + [A1 + G(A/G, 9\%, 6)] (F/A, 9\%, 6)$ = -5,00,000 (1.677) + [50,000 + 50,000(2.2498)] 7.523 = -8,38,500 + 12,22,412.27 = Rs. 3,83,912.27 Alternative 2 *P* = Rs. 7,00,000 A1 = Rs. 70,000 *G* = Rs. 70,000 *n* = 6 years The formula for the future worth of the alternative 2 is $FW_2(9\%) = -P(F/P, 9\%, 6) + [A1 + G(A/G, 9\%, 6)]$ (F/A, 9%, 6) = -7,00,000 1.677 + [70,000 + 70,000 2.2498] 7.523 = -11,73,900 + 17,11,377.18 = Rs. 5,37,477.18

The future worth of alternative 2 is more than that of alternative 1. Therefore, alternative 2 must be selected.

(ii) Evaluation at i = 20%: Alternative 1

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P = \text{Rs. } 5,00,000 \text{ } A1 = \text{Rs. } 50,000
G = \text{Rs. } 50,000 \text{ } n = 6 \text{ years}
The formula for the future worth of alternative 1 is
FW_1(20\%) = -P(F/P, 20\%, 6) + [A1 + G(A/G, 20\%, 6)] (F/A, 20\%, 6)
= -5,00,000(2.986) + [50,000 + 50,000(1.9788)] 9.93
= -14,93,000 + 14,78,974.20
= \text{Rs. } -14,025.80
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The negative sign of the future worth amount indicates that alternative 1 incurs loss.

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Alternative 2

P = \text{Rs. } 7,00,000

A1 = \text{Rs. } 70,000

G = \text{Rs. } 70,000 n = 6 \text{ years}

The formula for the future worth of alternative 2 is

FW_2(20\%) = -P(F/P, 20\%, 6) + [A1 + G(A/G, 20\%, 6)] (F/A, 20\%, 6)

= -7,00,000 2.986 + [70,000 + 70,000 1.9788] 9.93

= -20,90,200 + 20,70,563.88

= \text{Rs. } -19,636.12
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The negative sign of the above future worth amount indicates that alternative 2 incurs loss. Thus, none of the two alternatives should be selected.

EXAMPLE 5.4 M/S Krishna Castings Ltd. is planning to replace its annealing furnace. It has received tenders from three different original manufacturers of annealing furnace. The details are as follows.

	Manufacturer		
	1	2	3
Initial cost (Rs.)	80,00,000	70,00,000	90,00,000
Life (years)	12	12	12
Annual operation and maintenance cost (Rs.)	8,00,000	9,00,000	8,50,000
Salvage value after			
12 years	5,00,000	4,00,000	7,00,000
Which is the best altern	ative based on	future worth	method at $i = 20\%$

Solution Alternative 1—Manufacturer 1

First cost, *P* = Rs. 80,00,000

Life, n = 12 years

Annual operating and maintenance cost, A = Rs. 8,00,000 Salvage value at the end of furnace life = Rs. 5,00,000

The cash flow diagram for this alternative is shown in Fig. 5.9.



= Rs. 11,31,87,850

The future worth cost of alternative 2 is less than that of the other two alternatives. Therefore, M/s. Krishna castings should buy the annealing furnace from manufacturer 2.

EXAMPLE 5.5 A company must decide whether to buy machine A or machine B: Machine A Machine В Initial cost Rs. 4,00,000 Rs. 8,00,000 Useful life, in years 4 4 Salvage value at the end of machine life Rs. 2,00,000 Rs. 5,50,000 Annual maintenance cost Rs. 40,000 0 At 12% interest rate, which machine should be selected? (Use future worth method of comparison). Solution Machine A Initial cost of the machine, P = Rs. 4,00,000Life, n = 4 years Salvage value at the end of machine life, S = Rs. 2,00,000Annual maintenance cost, A = Rs. 40,0002,00,000 Interest rate, *i* = 12%, compounded annually. *i* = 20% The cash flow diagram of machine A is given in Fig. 5.12. 4 40,000 40,000 40,000 40,000 Fig. 5.12 Cash flow diagram for machine A. 4,00,000 The future worth function of Fig. 5.12 is $FW_A(12\%) = 4,00,000 \ (F/P, 12\%, 4) + 40,000 \ (F/A, 12\%, 4) - 2,00,000$ = 4,00,000 (1.574) + 40,000 (4.779) - 2,00,000 = Rs. 6,20,760 Machine B Initial cost of the machine, P = Rs. 8,00,000Life, n = 4 years Salvage value at the end of machine life, S = Rs. 5,50,000 Annual maintenance cost, A = zero. Interest rate, *i* = 12%, compounded annually. The cash flow diagram of the machine *B* is illustrated in Fig. 5.13. 5,50,000 *i* = 12% 2 0 8,00,000 Fig. 5.13 Cash flow diagram for machine *B*. The future worth function of Fig 5.13 is $FW_B(12\%) = 8,00,000 (F/P, 12\%, 4) - 5,50,000$

= 8,00,000 (1.574) - 5,50,000 = Rs. 7,09,200

The future worth cost of machine A is less than that of machine B. Therefore, machine A should be selected.

5.6 Review Questions

1. A suburban taxi company is considering buying taxis with diesel engines instead of petrol engines. The cars average 50,000 km a year, with a useful life of three years for the taxi with the petrol engine and four years for the diesel taxi. Other comparative information are as follows:

	Diesel	Petrol
Vehicle cost	Rs. 5,00,000	Rs.
		4,00,000
Fuel cost per litre	Rs. 9.00	Rs. 24.00
Mileage, in km/litre	30	20
Annual insurance premium	Rs. 500	Rs. 500
Salvage value at the end		
of vehicle life	Rs. 70,000	Rs.
		1,00,000

Determine the more economical choice based on the future worth method of comparison if the interest rate is 15%, compounded annually.

- 2. A motorcycle is sold for Rs. 50,000. The motorcycle dealer is willing to sell it on the following terms:
 - (a) Make no down payment but pay Rs. 1,500 at the end of each of the firstfour months and Rs. 3,000 at the end of each month after that for 18 continuous months.
 - (b) Make no down payment but pay a total amount of Rs. 90,000 at the endof the 22nd month; till that time the buyer should mortgage property worth of Rs. 50,000, at present.

Based on these terms and a 12% annual interest rate compounded monthly, find the best alternative for the buyer based on the future worth method of comparison.

3. Consider the following two mutually exclusive alternatives.

	A	B
Cost	Rs. 4,000	Rs.
		6,000
Uniform annual benefit	Rs. 640	Rs.
		960
Useful life (years)	20	20

Using a 15% interest rate, determine which alternative should be selected based on the future worth method of comparison.

4. A company must decide whether to buy machine *A* or machine *B*:

	Machine A	Machine B
Initial cost	Rs. 4,00,000	Rs. 8,00,000
Useful life, (years)	5	5
Salvage value at the end		
of machine life	Rs. 2,00,000	Rs. 5,50,000
Annual maintenance cost	Rs. 40,000	0
		1 . 10 (

At 15% interest rate, which machine should be selected? (Use the future worth method of comparison.)

5. Due to increasing awareness of customers, two different television manufacturing companies started a marketing war. The details of advertisements of the companies are as follows:

	Brand X	Brand Y
Selling price of a TV set	Rs. 15,000	Rs.
		10,000
Amount returned to buyer after 5	Rs. 8,000	_
years		

Select the most economical brand from the customer's point of view using the future worth method of comparison, assuming an interest rate of 15%, compounded annually.

- 6. Alpha Finance Company is coming with an option of accepting Rs. 10,000 now and paying a sum of Rs. 1,60,000 after 20 years. Beta Finance Company is coming with a similar option of accepting Rs. 10,000 now and paying a sum of Rs. 3,00,000 after 25 years. Compare and select the best alternative based on the future worth method of comparison with 15% interest rate, compounded annually.
- 7. An insurance company gives an endowment policy for a person aged 30 years. The yearly premium for an insured sum of Rs. 1,00,000 is Rs. 4,000. The policy will mature after 25 years. Also, the person is entitled for a bonus of Rs. 75 per thousand per year at the end of the policy. If a person survives till the end of the 25th year:
 - (a) What will be the total sum that he will get from the insurance companyat that time?
 - (b) Instead of paying the premiums for the insurance policy, if the personinvests an equal sum of Rs.
 4,000 at the end of each year for the next 25 years in some other scheme which is having similar tax benefit, find the future worth of the investment at 15% interest rate, compounded annually.
 - (c) Rate the above alternatives assuming that the person is sure of livingfor the next 25 years.