

5 TESTS OF HYPOTHESES CONCEPT

5.1 POPULATION PARAMETERS: Population parameter is a term used in statistics. It refers to a measurement of a population that is being studied. The parameter is a number that is used by scientists to describe a specific population or group.

Populations

A population is any group of people or objects that a researcher wants to measure. Examples of populations are people in the United States, or trees in a forest.

Unit

A population parameter must also have a unit that is being measured. For people in the United States, possible units that could be measured include number, height and eye color.

Population Size

Population parameters can refer to very large or very small populations. Small populations can be measured directly, but when researchers study large populations, they often take samples to represent the entire population.

Statistics

Statistics is a branch of mathematics that allows researchers to make guesses about population parameters based on data gathered from random samples of a population. Depending on the size of the sample, researchers will use a margin of error to describe how close they think a sample statistic is to a population parameter.

A **statistical parameter** is a parameter that indexes a family of probability distributions. It can be regarded as a numerical characteristic of a population or a model.^[1]

Among parameterized families of distributions are the normal distributions, the Poisson distributions, the binomial distributions, and the exponential distributions. The family of normal distributions has two parameters, the mean and the variance: if these are specified, the distribution is known exactly. The family of chi-squared distributions, on the other hand, has only one parameter, the number of degrees of freedom.

In statistical inference, parameters are sometimes taken to be unobservable, and in this case the statistician's task is to infer what they can about the parameter based on observations of random variables distributed according to the probability

distribution in question, or, more concretely stated, based on a random sample taken from the population of interest. In other situations, parameters may be fixed by the nature of the sampling procedure used or the kind of statistical procedure being carried out (for example, the number of degrees of freedom in a Pearson's chi-squared test).

Even if a family of distributions is not specified, quantities such as the mean and variance can still be regarded as parameters of the distribution of the population from which a sample is drawn. Statistical procedures can still attempt to make inferences about such population parameters. Parameters of this type are given names appropriate to their roles, including:

- location parameter
- dispersion parameter or scale parameter
- shape parameter

Where a probability distribution has a domain over a set of objects that are themselves probability distributions, the term concentration parameter is used for quantities that index how variable the outcomes would be.

Quantities such as regression coefficients, are statistical parameters in the above sense, since they index the family of conditional probability distributions that describe how the dependent variables are related to the independent variables.

5.2 BASIC STEPS IN HYPOTHESIS TESTING: Hypothesis testing is a way of systematically quantifying how certain you are of the result of a statistical experiment.

The Null Hypothesis

The most common type of hypothesis testing involves a *null hypothesis*. The null hypothesis, denoted H_0 , is a statement about the world which can plausibly account for the data you observe. Don't read anything into the fact that it's called the "null" hypothesis — it's just the hypothesis we're trying to test.

For example, "the coin is fair" is an example of a null hypothesis, as is "the coin is biased." The important part is that the null hypothesis be able to be expressed in simple, mathematical terms. We'll see how to express these statements mathematically in just a bit.

The main goal of hypothesis testing is to tell us whether we have enough evidence to reject the null hypothesis. In our case we want to know whether the coin is biased or not, so our null hypothesis should be "the coin is fair." If we get enough evidence that contradicts this hypothesis, say, by flipping it 100 times and having it come up heads only once, then we can safely reject it.

All of this is perfectly quantifiable, of course. What constitutes "enough" and "safely" are all a matter of statistics.

The Statistics, Intuitively

So, we have a coin. Our null hypothesis is that this coin is fair. We flip it 100 times and it comes up heads 51 times. Do we know whether the coin is biased or not?

Our gut might say the coin is fair, or at least probably fair, but we can't say for sure. The expected number of heads is 50 and 51 is quite close. But what if we flipped the coin 100,000 times and it came up heads 51,000 times? We see 51% heads both times, but in the second instance the coin is more likely to be biased.

Lack of evidence to the contrary is not evidence that the null hypothesis is true. Rather, it means that we don't have sufficient evidence to conclude that the null hypothesis is false. The coin might actually have a 51% bias towards heads, after all.

If instead we saw 1 head for 100 flips that would be another story. Intuitively we know that the chance of seeing this if the null hypothesis were true is so small that we would be comfortable rejecting the null hypothesis and declaring the coin to (probably) be biased.

5.3 STEPS OF HYPOTHESIS TESTING:

- The first step is to specify the *null hypothesis*. For a *two-tailed* test, the null hypothesis is typically that a parameter equals zero although there are exceptions. A typical null hypothesis is $\mu_1 - \mu_2 = 0$ which is equivalent to $\mu_1 = \mu_2$. For a *one-tailed* test, the null hypothesis is either that a parameter is greater than or equal to zero or that a parameter is less than or equal to zero. If the prediction is that μ_1 is larger than μ_2 , then the null hypothesis (the reverse of the prediction) is $\mu_2 - \mu_1 \geq 0$. This is equivalent to $\mu_1 \leq \mu_2$.

- The second step is to specify the α level which is also known as the significance level. Typical values are 0.05 and 0.01.
- The third step is to compute the *probability value* (also known as the p value). This is the probability of obtaining a sample statistic as different or more different from the parameter specified in the null hypothesis given that the null hypothesis is true.
- Finally, compare the probability value with the α level. If the probability value is lower then you reject the null hypothesis. Keep in mind that rejecting the null hypothesis is not an all-or-none decision. The lower the probability value, the more confidence you can have that the null hypothesis is false. However, if your probability value is higher than the conventional α level of 0.05, most scientists will consider your findings inconclusive. Failure to reject the null hypothesis does not constitute support for the null hypothesis. It just means you do not have sufficiently strong data to reject it.

5.4 BASIC PROCEDURES FOR HYPOTHESIS TESTING: A hypothesis is a statement or a tentative theory that may or may not be true, but is initially assumed to be true until new evidence suggests otherwise. It may be proposed from a preliminary observation, a guess or based from previous experiences. In hypothesis testing problem, the researcher has in mind a specific notion concerning the characteristics of the population under study before the sample data are gathered. Then investigate the sample information to examine how consistent the data with the hypothesis in questioned.

If the sample information deviate much from the stated hypothesis, then researcher tend to disbelieved and reject the proposed statement. Although the proposed statement may be true, it is expected that any single sample (or samples) will differ slightly from the true characteristic of the population and other will not, because of the sampling variation, have the same exact value as the population parameter. Hence, differences between the sample information and the population under study might be due chance. The procedure of statistical test will provide the basis in deciding whether differences between the sample observation and the hypothesized

value could be due to sampling variation alone, or are so large enough as to make the proposed statement untenable.

Types of Hypothesis

Null hypothesis – the null hypothesis is denoted by H_0 , it is the hypothesis of “no difference” and usually formulated for the purpose of being rejected. Alternative hypothesis – the alternative hypothesis is denoted by H_a or H_1 . This is the hypothesis that contradicts the null hypothesis. If the null hypothesis is rejected, the alternative is being supported. The alternative hypothesis is the operational statement of the experimenter’s research hypothesis

5.5 TYPE I AND TYPE II ERRORS: In all tests of hypothesis, there are two types of errors that can be committed. The first is called a Type I error and refers to the situation where we incorrectly reject H_0 when in fact it is true. This is also called a false positive result (as we incorrectly conclude that the research hypothesis is true when in fact it is not). When we run a test of hypothesis and decide to reject H_0 (e.g., because the test statistic exceeds the critical value in an upper tailed test) then either we make a correct decision because the research hypothesis is true or we commit a Type I error. The different conclusions are summarized in the table below. Note that we will never know whether the null hypothesis is really true or false.

5.6 HYPOTHESIS TESTING: When you conduct a piece of quantitative research, you are inevitably attempting to answer a research question or hypothesis that you have set. One method of evaluating this research question is via a process called **hypothesis testing**, which is sometimes also referred to as **significance testing**.

The first step in hypothesis testing is to set a research hypothesis. In an example: Sarah and Mike are teachers. The aim is to examine the effect that two different teaching methods – providing both lectures and seminar classes (Sarah), and providing lectures by themselves (Mike) – had on the performance of Sarah's 50 students and Mike's 50 students. More specifically, they want to determine whether performance is different between the two different teaching methods. Whilst Mike is skeptical about the effectiveness of seminars, Sarah clearly believes that giving seminars in addition to lectures helps her students do better than those in Mike's class. This leads to the following research hypothesis:

Research Hypothesis:

When students attend seminar classes, in addition to lectures, their performance increases.

Before moving onto the second step of the hypothesis testing process, I need to explain why you need to run hypothesis testing at all.

Sample to population

If you have measured individuals (or any other type of "object") in a study and want to understand differences (or any other type of effect), you can simply summarize the data you have collected. For example, if Sarah and Mike wanted to know which teaching method was the best, they could simply compare the performance achieved by the two groups of students – the group of students that took lectures and seminar classes, and the group of students that took lectures by themselves – and conclude that the best method was the teaching method which resulted in the highest performance. However, this is generally of only limited appeal because the conclusions could only apply to students in this study. However, if those students were representative of all statistics students on a graduate management degree, the study would have wider appeal.

In statistics terminology, the students in the study are the **sample** and the larger group they represent (i.e., all statistics students on a graduate management degree) is called the **population**. Given that the sample of statistics students in the study are representative of a larger population of statistics students, you can use hypothesis testing to understand whether any differences or effects discovered in the study exist in the population. In layman's terms, hypothesis testing is used to establish whether a research hypothesis extends beyond those individuals examined in a single study.

Another example could be taking a sample of 200 breast cancer sufferers in order to test a new drug that is designed to eradicate this type of cancer. As much as you are interested in helping these specific 200 cancer sufferers, your real goal is to establish that the drug works in the population (i.e., all breast cancer sufferers).

As such, by taking a hypothesis testing approach, Sarah and Mike want to **generalize** their results to a population rather than just the students in their sample. However, in order to use hypothesis testing, you need to re-state your research hypothesis as a null and alternative hypothesis. Before you can do this, it

is best to consider the process/structure involved in hypothesis testing and what you are measuring.

The structure of hypothesis testing

Whilst all pieces of quantitative research have some dilemma, issue or problem that they are trying to investigate, the focus in hypothesis testing is to find ways to structure these in such a way that we can test them effectively. Typically, it is important to:

1. Define the **research hypothesis** for the study.
2. Explain how you are going to **operationalize** (that is, **measure** or **operationally define**) what you are studying and set out the **variables** to be studied.
3. Set out the **null** and **alternative hypothesis** (or more than one hypothesis; in other words, a number of hypotheses).
4. Set the **significance level**.
5. Make a **one-** or **two-tailed prediction**.
6. Determine whether the distribution that you are studying is **normal** (this has implications for the types of statistical tests that you can run on your data).
7. **Select** an appropriate **statistical test** based on the variables you have defined and whether the distribution is normal or not.
8. **Run** the **statistical tests** on your data and **interpret** the **output**.
9. **Reject** or **fail to reject** the **null hypothesis**.