THE PARTICLE IV

4.1Mechanical equilibrium: A mechanical equilibrium is a state in which a momentum coordinate of a particle, rigid body, or dynamical system is conserved. Usually this refers to linear momentum. For instance, a linear mechanical equilibrium would be a state in which the linear momentum of the system is conserved at the net force on the object is zero. In the specific case that the linear momentum is zero and conserved, the system can be said to be in a static equilibrium. Of course, in any system is conserved linear momentum, it is possible to shift to a non-inertial reference frame that is stationary with respect to the object.

In a rotational mechanical equilibrium the angular momentum of the object is conserved and the net torque is zero. More generally in conservative systems, equilibrium is established at a point in configuration space where the gradient with respect to the generalized coordinates of the potential energy is zero.

Stability

An important property of systems at mechanical equilibrium is their stability. In the terminology of elementary calculus, a system at mechanical equilibrium is at a critical point in potential energy where the first derivative is zero. To determine whether or not the system is stable or unstable, we apply the second derivative test:

Unstable equilibria

Second derivative < 0: The potential energy is at a local maximum, which means that the system is in an unstable equilibrium state. If the system is displaced an arbitrarily small distance from the equilibrium state, the forces of the system cause it to move even farther away.

Stable equilibria

Second derivative > 0: The potential energy is at a local minimum. This is a stable equilibrium. The response to a small perturbation is forces that tend to restore the equilibrium. If more than one stable equilibrium state is possible for a system, any equilibria whose potential energy is higher than the absolute minimum represent metastable states.

Neutral equilibria

> Second derivative = 0 or does not exist: The state is neutral to the lowest order and nearly remains in equilibrium if displaced a small amount. To investigate the precise stability of the system, higher order derivatives must be examined. The state is unstable if the lowest nonzero derivative is of odd order or has a negative value, stable if the lowest nonzero derivative is both of even order and has a positive value, and neutral if all higher order derivatives are zero. In a truly neutral state the energy does not vary and the state of equilibrium has a finite width. This is sometimes referred to as state that is marginally stable or in a state of indifference.

When considering more than one dimension, it is possible to get different results in different directions, for example stability with respect to displacements in the *x*-direction but instability in the *y*-direction, a case known as a saddle point. Generally an equilibrium is only referred to as stable if it is stable in all directions.

Examples

The special case of mechanical equilibrium of a stationary object is static equilibrium. A paperweight on a desk would be in static equilibrium. The minimal number of static equilibria of homogeneous, convex bodies (when resting under gravity on a horizontal surface) is of special interest. In the planar case, the minimal number is 4, while in three dimensions one can build an object with just one stable and one unstable balance point, this is called gomboc. A child sliding down a slide at constant speed would be in mechanical equilibrium, but not in static equilibrium (in the reference frame of the slide).

An example of mechanical equilibrium is a person trying to press a spring. He or she can push it up to a point after which it reaches a state where the force trying to compress it and the resistive force from the spring are equal, so the person cannot further press it. At this state the system will be in mechanical equilibrium. When the pressing force is removed the spring attains its original state.

Dynamic Equilibrium

A dynamic equilibrium exists once a reversible reaction ceases to change its ratio of reactants/products, but substances move between the chemicals at an equal rate, meaning there is no net change. It is a particular example of a system in a steady state. In thermodynamics a closed system is in thermodynamic equilibrium when reactions occur at such rates that the composition of the mixture does not change with time. Reactions do in fact occur, sometimes vigorously, but to such an extent that changes in composition cannot be observed. Equilibrium constants can be expressed in terms of the rate constants for elementary reactions.

Examples

In a new bottle of cola the concentration of carbon dioxide in the liquid phase has a particular value. If half of the liquid is poured out and the bottle is sealed, carbon dioxide will leave the liquid phase at an ever decreasing rate and the partial pressure of carbon dioxide in the gas phase will increase until equilibrium is reached. At that point, due to thermal motion, a molecule of CO2 may leave the liquid phase, but within a very short time another molecule of CO2 will pass from the gas to the liquid, and vice-versa. At equilibrium the rate of transfer of CO2 from the gas to the liquid phase is equal to the rate from liquid to gas. In this case, the equilibrium concentration of CO2 in the liquid is given by Henry's law, which states that the solubility of a gas in a liquid is directly proportional to the partial pressure of that gas above the liquid. This relationship is written as: $c = kp$,

Where k is a temperature-dependent constant, p is the partial pressure and c is the concentration of the dissolved gas in the liquid. Thus, the partial pressure of CO2 in the gas has increased until Henry's law is obeyed. The concentration of carbon dioxide in the liquid has decreased and the drink has lost some of its fizz.

Henry's law may be derived by setting the chemical potentials of carbon dioxide in the two phases to be equal to each other. Equality of chemical potential defines chemical equilibrium. Other constants for dynamic equilibrium involving phase changes include partition coefficient and solubility product. Raoult's law defines the equilibrium vapor pressure of an ideal solution.

Dynamic equilibria can also exist in a single-phase system. A simple example occurs with acid-base equilibria such as the "dissociation" of acetic acid, in aqueous solution.

4.2 Euclidean vector

In mathematics, physics, and engineering, a **Euclidean vector** (sometimes called a **geometric** or **spatial vector**, or—as here—simply a **vector**) is a geometric quantity having magnitude (or length) and direction expressed numerically as tuples [x, y, z] splitting the entire quantity into its orthogonal-axis components. **A vector is an object that is an input for or an output from vector functions according to vector algebra.**

A Euclidean vector is typically sketched as a directed line segment, or arrow, connecting an *initial point A* with a *terminal point B*and denoted by $\overrightarrow{AB}$. However, as an informational object, the vector is not as informative as a directed line segment (an ordered list of two points [A, B]) but rather expresses the displacement, or vector offset (change in location), A → B. Technically, the [x, y, z] components of vector $\overrightarrow{AB}$ are equal to the vector difference $\overrightarrow{B}$ minus $\overrightarrow{A}$. In this way, the vector $\overrightarrow{AB}$considered as a numerical quantity conceals the locations of A and B while imparting the location of point B relative to A as if A were the coordinate origin.

Vectors play an important role in physics: velocity and acceleration of a moving object and forces acting on it are all described by vectors. Many other physical quantities can be usefully thought of as vectors. Although most of them do not represent distances (except, for example, position or displacement), their magnitude and direction can be still represented by the length and direction of an arrow. The mathematical representation of a physical vector depends on the coordinate system used to describe it. Other vector-like objects that describe physical quantities and transform in a similar way under changes of the coordinate system include pseudovectors and tensors.

It is important to distinguish Euclidean vectors from the more general concept in linear algebra of vectors as elements of a vector space. General vectors in this sense are fixed-size, ordered collections of items as in the case of Euclidean vectors, but the individual items may not be real numbers, and the normal Euclidean concepts of length, distance and angle may not be applicable. (A vector space with a definition of these concepts is called an inner product space.) In turn, both of these definitions of vector should be distinguished from the statistical concept of a random vector. The individual items in a random vector are individual real-valued random variables, and are often manipulated using the same sort of mathematical vector and matrix operations that apply to the other types of vectors, but otherwise usually behave more like collections of individual values. Concepts of length, distance and angle do not normally apply to these vectors, either; rather, what links the values together is the potential correlations among them.

The word "vector" originates from the Latin *vehere* meaning "to carry". It was first used by 18th century astronomers investigating planet rotation around the Sun.

History

The concept of vector, as we know it today, evolved gradually over a period of more than 200 years. About a dozen people made significant contributions.The immediate predecessor of vectors were quaternions, devised by William Rowan Hamilton in 1843 as a generalization of complex numbers. Initially, his search was for a formalism to enable the analysis of three-dimensional space in the same way that complex numbers had enabled analysis of two-dimensional space, but he arrived at a four-dimensional system. In 1846 Hamilton divided his quaternions into the sum of real and imaginary parts that he respectively called "scalar" and "vector":

The algebraically imaginary part, being geometrically constructed by a straight line, or radius vector, which has, in general, for each determined quaternion, a determined length and determined direction in space, may be called the vector part, or simply the vector of the quaternion.

Several other mathematicians developed vector-like systems around the same time as Hamilton including Giusto Bellavitis, Augustin Cauchy, Hermann Grassmann, August Möbius, Comte de Saint-Venant, and Matthew O'Brien. Grassmann's 1840 work Theorie der Ebbe und Flut (Theory of the Ebb and Flow) was the first system of spatial analysis similar to today's system and had ideas corresponding to the cross product, scalar product and vector differentiation. Grassmann's work was largely neglected until the 1870s.

Peter Guthrie Tait carried the quaternion standard after Hamilton. His 1867 Elementary Treatise of Quaternions included extensive treatment of the nabla or del operator ∇.

In 1878 Elements of Dynamic was published by William Kingdon Clifford. Clifford simplified the quaternion study by isolating the dot product and cross product of two vectors from the complete quaternion product. This approach made vector calculations available to engineers and others working in three dimensions and skeptical of the fourth.

Josiah Willard Gibbs, who was exposed to quaternions through James Clerk Maxwell's Treatise on Electricity and Magnetism, separated off their vector part for independent treatment. The first half of Gibbs's Elements of Vector Analysis, published in 1881, presents what is essentially the modern system of vector analysis. In 1901 Edwin Bidwell Wilson published Vector Analysis, adapted from Gibb's lectures, and banishing any mention of quaternions in the development of vector calculus.

Overview

In physics and engineering, a vector is typically regarded as a geometric entity characterized by a magnitude and a direction. It is formally defined as a directed line segment, or arrow, in a Euclidean space. In pure mathematics, a vector is defined more generally as any element of a vector space. In this context, vectors are abstract entities which may or may not be characterized by a magnitude and a direction. This generalized definition implies that the above-mentioned geometric entities are a special kind of vectors, as they are elements of a special kind of vector space called Euclidean space.

This article is about vectors strictly defined as arrows in Euclidean space. When it becomes necessary to distinguish these special vectors from vectors as defined in pure mathematics, they are sometimes referred to as geometric, spatial, or Euclidean vectors.

Being an arrow, a Euclidean vector possesses a definite initial point and terminal point. A vector with fixed initial and terminal point is called a bound vector. When only the magnitude and direction of the vector matter, then the particular initial point is of no importance, and the vector is called a free vector. Thus two arrows \overrightarrow{AB} and \overrightarrow{A'B'} in space represent the same free vector if they have the same magnitude and direction: that is, they are equivalent if the quadrilateral ABB′A′ is a parallelogram. If the Euclidean space is equipped with a choice of origin, then a free vector is equivalent to the bound vector of the same magnitude and direction whose initial point is the origin.

The term vector also has generalizations to higher dimensions and to more formal approaches with much wider applications.

Examples in one dimension

Since the physicist's concept of force has a direction and a magnitude, it may be seen as a vector. As an example, consider a rightward force F of 15 newtons. If the positive axis is also directed rightward, then F is represented by the vector 15 N, and if positive points leftward, then the vector for F is −15 N. In either case, the magnitude of the vector is 15 N. Likewise, the vector representation of a displacement Δs of 4 meters to the right would be 4 m or −4 m, and its magnitude would be 4 m regardless.

In physics and engineering

Vectors are fundamental in the physical sciences. They can be used to represent any quantity that has magnitude, has direction, and which adheres to the rules of vector addition. An example is velocity, the magnitude of which is speed. For example, the velocity 5 meters per second upward could be represented by the vector (0,5) (in 2 dimensions with the positive y axis as 'up'). Another quantity represented by a vector is force, since it has a magnitude and direction and follows the rules of vector addition. Vectors also describe many other physical quantities, such as linear displacement, displacement, linear acceleration, angular acceleration, linear momentum, and angular momentum. Other physical vectors, such as the electric and magnetic field, are represented as a system of vectors at each point of a physical space; that is, a vector field. Examples of quantities that have magnitude and direction but fail to follow the rules of vector addition: Angular displacement and electric current. Consequently, these are not vectors