

## TIMES AND EQUIVALENT SYSTEMS

### 5.1 EQUIVALENT SYSTEMS OF FORCES:

#### Introduction

In general it is not feasible to consider bodies as point objects while considering the effect of forces. For tackling practical problems we need to take the dimension of bodies into account. A body which doesn't deform by the application of force is termed as rigid bodies. Actual structures and machines, however, are never absolutely rigid and deform under the loads to which they are subjected.

In the following unit we will be dealing with principle of transmissibility of forces, moment of a force about a point, moment of a force about an axis, moment of a couple, addition of couple moments, resolution of a force into a force couple system, resolution of a system of force into a force couple system, resolution of a system of forces into wrench.

Letters in bold indicate vectors.

#### Principle of transmissibility, sliding vectors

Let's assume that a force  $F$  acts at on the rigid body. Let's say that the force  $F'$  has the same magnitude and direction and acts along the same line of action as that of  $F$ . Principle of transmissibility states that without altering the equations of equilibrium or motion and provided the above conditions are satisfied, the force  $F$  can be equivalently replaced by force  $F'$ . Therefore the force acting on the rigid body can be termed as a sliding vector, which is allowed to slide along its line of action.

#### Moment of a force

#### MOMENT OF A FORCE ABOUT A POINT

Effect of a force not only depends on its magnitude and direction, but also depends on its point of application. Force is responsible for translational as well as rotational motion of a object. Let a force  $F$  be applied at a point  $A$  as

shown in figure above. Position vector of A with respect to B is given by  $r_{AB}$ . Therefore moment of the force  $F$  at A with respect to point B is given by:  $M_B = r_{AB} \times F$ . The magnitude of  $M_B$  is a measurement of rotating tendency of the object about B. Its SI unit is Newton-metre (Nm).

### Points to note:

$M_B$  is a vector perpendicular to the plane containing  $r_{AB}$  and  $F$ . Direction determined by right hand thumb rule. Rotation is anti-clockwise if  $M_B$  points outward to the plane and vice-versa.  $|M_B| = |F||r_{AB}| \sin\alpha$ , where  $\alpha$  is the angle between force and position vector.

In case of multiple forces acting at point B, moment of the resultant force is given by sum of individual moment of forces about point B.  $M_B = r \times (F_1 + F_2 + F_3 + \dots) = r \times F_1 + r \times F_2 + r \times F_3 + \dots$ . For an object with zero rotational motion the net moment of forces about every point should be zero.

## MOMENT OF A FORCE ABOUT AN AXIS

Earlier we had seen that  $M_O = r_A \times F$ . Now we define another useful term which is moment about an axis. It is given by:  $M_P = \hat{i} \cdot (r_A \times F)$  where  $\hat{i}$  is the unit vector in the direction of the axis OP. Moment about an axis measures the tendency of the force to rotate the body along the given axis.

### Points to note:

It is a scalar triple product which gives a scalar output. Result is independent of the point selected on the axis i.e.  $r_{BA}$  can be used instead of  $r_A$ . To solve problems most convenient position vector should be selected in order to simplify calculations. For an object with zero rotational motion the net moment of forces about every axis should be zero.

### Moment of a couple

A couple system consists of two parallel forces equal in magnitude but opposite in direction. The above diagram shows the exact situation. Since the algebraic sum of force vectors is zero, the force will produce no translational effect on the body. But the body will tend to rotate in effect of the two forces. The derived result for moment due to couple turns out to be:  $M_O = r \times F$  Where  $F$  is the force vector and  $r$  is the vector joining points of application of two

forces. We also get,  $|\mathbf{M}_O| = |\mathbf{F}|r \sin \alpha = |\mathbf{F}|d$ , where  $\alpha$  is the angle between the two vectors, where 'd' is the perpendicular distance between two parallel force vectors (as shown in figure above).

### Points to note:

Since the results turn out to be independent of origin the same result would have been obtained if the origin was shifted. Therefore couple moment is a free vector which can be applied at any point. Rotation is anti-clockwise if moment vector points outward to the plane of forces and vice versa. Since couple moments are vector quantities, it is derivable that individual couple moments acting upon a rigid body can be added up vectorially. Two sets of couples producing same couple moment in the same direction are said to be equivalent couples. Couple moments are represented as vectors pointing outwards or inwards as the case may be.

### Resolution of a force into a force couple system

In the above situation we have a force  $\mathbf{F}$  acting in the rigid body at a point N. The principle of transmissibility does not allow us to shift forces parallel. To shift the force parallel we can add two forces at the point M equal in magnitude with the force  $\mathbf{F}$  but opposite in direction directed parallel to initial force  $\mathbf{F}$ . Hence we are in a situation where we can replace the two forces  $\mathbf{F}$  and  $-\mathbf{F}$  by a single couple moment  $\mathbf{M}_O$  emerging out of the plane at the point M along with a shifted force vector. Thus, we can shift a force vector parallel to a point by adding a moment vector at a moment vector, created by the original force about the point.

We can state that a force couple system about a point can be equivalently replaced by a single force. For this we shift the force parallel until we have the moment of the force about the given point equal in magnitude and opposite in sense to the moment which is to be eliminated.

### Equivalent Systems of forces

We have just shown that a single force could be shifted parallel to a itself to a specific point by addition of a couple moment. This statement can be generalized for a system of forces acting on a system.

Drawing parallel to the last topic we shift each individual force at 'O' and add moments created by the forces at 'O'. Finally we add up the vectors.

Resultant force at O:  $F_R = F_1 + F_2 + F_3$

Resultant moment at O:  $M_O = r_1 \times F_1 + r_2 \times F_2 + r_3 \times F_3$

Thus, every complex system of forces can be thus reduced to a simple Couple – Force equivalent system. Unlike the previous case, we may not have resultant moment vector perpendicular to the resultant force vector. The equivalent force couple system is a characteristic of the system. Two systems are said to be equivalent if they reduce to the same force couple system at the same point.

### Reduction of systems of forces to Wrench

Any complex system of forces can be reduced to an equivalent force couple system as shown below:

$M_O$  is broken into two perpendicular components  $M_a$  and  $M_p$ , one along  $F_r$  and other perpendicular to  $F_r$  as shown in figure. We shift  $F_r$  parallel to a point Y such that:  $M_p = -(r_{YX} \times R)$ . Thus the additional moment added to the system on shifting of the force  $F_r$  cancels  $M_p$ . Since  $M_a$  is a free vector it can also be shifted to the point A. This particular force-couple system is called a wrench because the resulting combination of push and twist is the same as that would be caused by an actual wrench. The axis of force passing through Y is called the axis of a wrench. Pitch is defined as the ratio of:  $[|M_a| : |F_r|]$   
 Pitch is also given as  $= M_O \cdot F_r / |F_r|^2$   
 Axis = (pitch) \*  $M_O$

**5.2 TIME OF A FORCE ABOUT A POINT:** In physics, a force is any influence which tends to change the motion of an object. In other words, a force can cause an object with mass to change its velocity (which includes to begin moving from a state of rest), i.e., to accelerate. Force can also be described by intuitive concepts such as a push or a pull. A force has both magnitude and direction, making it a vector quantity. It is measured in the SI unit of newtons and represented by the symbol  $F$ .

The original form of Newton's second law states that the net force acting upon an object is equal to the rate at which its momentum changes with time. If the mass of

the object is constant, this law implies that the acceleration of an object is directly proportional to the net force acting on the object, is in the direction of the net force, and is inversely proportional to the mass of the object. As a formula, this is expressed as:

$$\vec{F} = m\vec{a}$$

where the arrows imply a vector quantity possessing both magnitude and direction.

Related concepts to force include: thrust, which increases the velocity of an object; drag, which decreases the velocity of an object; and torque which produces changes in rotational speed of an object. In an extended body, each part usually applies forces on the adjacent parts; the distribution of such forces through the body is the so-called mechanical stress. Pressure is a simple type of stress. Stress usually causes deformation of solid materials, or flow in fluids

### Development of the concept

Philosophers in antiquity used the concept of force in the study of stationary and moving objects and simple machines, but thinkers such as Aristotle and Archimedes retained fundamental errors in understanding force. In part this was due to an incomplete understanding of the sometimes non-obvious force of friction, and a consequently inadequate view of the nature of natural motion. A fundamental error was the belief that a force is required to maintain motion, even at a constant velocity. Most of the previous misunderstandings about motion and force were eventually corrected by Sir Isaac Newton; with his mathematical insight, he formulated laws of motion that were not improved-on for nearly three hundred years. By the early 20th century, Einstein developed a theory of relativity that correctly predicted the action of forces on objects with increasing momenta near the speed of light, and also provided insight into the forces produced by gravitation and inertia.

With modern insights into quantum mechanics and technology that can accelerate particles close to the speed of light, particle physics has devised a Standard Model to describe forces between particles smaller than atoms. The Standard Model predicts that exchanged particles called gauge bosons are the fundamental means by which forces are emitted and absorbed. Only four main interactions are known: in order of decreasing strength, they are: strong, electromagnetic, weak, and gravitational. High-energy particle physics observations made during the 1970s

and 1980s confirmed that the weak and electromagnetic forces are expressions of a more fundamental electroweak interaction.

## Descriptions

Diagrams of a block on a flat surface and an inclined plane. Forces are resolved and added together to determine their magnitudes and the net force.

Since forces are perceived as pushes or pulls, this can provide an intuitive understanding for describing forces. As with other physical concepts (e.g. temperature), the intuitive understanding of forces is quantified using precise operational definitions that are consistent with direct observations and compared to a standard measurement scale. Through experimentation, it is determined that laboratory measurements of forces are fully consistent with the conceptual definition of force offered by Newtonian mechanics.

Forces act in a particular direction and have sizes dependent upon how strong the push or pull is. Because of these characteristics, forces are classified as "vector quantities". This means that forces follow a different set of mathematical rules than physical quantities that do not have direction (denoted scalar quantities). For example, when determining what happens when two forces act on the same object, it is necessary to know both the magnitude and the direction of both forces to calculate the result. If both of these pieces of information are not known for each force, the situation is ambiguous. For example, if you know that two people are pulling on the same rope with known magnitudes of force but you do not know which direction either person is pulling, it is impossible to determine what the acceleration of the rope will be. The two people could be pulling against each other as in tug of war or the two people could be pulling in the same direction. In this simple one-dimensional example, without knowing the direction of the forces it is impossible to decide whether the net force is the result of adding the two force magnitudes or subtracting one from the other. Associating forces with vectors avoids such problems.