TIMES AND EQUIVALENT SYSTEMS II

6.1 for concurrent forces Varignon theorem: Varignon's theorem is a statement in Euclidean geometry by Pierre Varignon that was first published in 1731. It deals with the construction of a particular parallelogram (**Varignon parallelogram**) from an arbitrary quadrangle.

> *The midpoints of the sides of an arbitrary quadrangle form a parallelogram. If the quadrangle is convex or reentrant, i.e. not a crossing quadrangle, then the area of the parallelogram is half as big as the area of the quadrangle.*

If one introduces the concept of oriented areas for n-gons, then the area equality above holds for crossed quadrangles as well. The Varignon parallelogram exists even for a skew quadrilateral, and is planar whether or not the quadrilateral is planar. It can be generalized to the midpoint polygon of an arbitrary polygon.

Special cases

The Varignon parallelogram is a rhombus if and only if the two diagonals of the quadrilateral have equal length, that is, if the quadrilateral is an equidiagonal quadrilateral.

The Varignon parallelogram is a rectangle if and only if the diagonals of the quadrilateral are perpendicular, that is, if the quadrilateral is an orthodiagonal quadrilateral.

Proof

Varignon's theorem is easily proved as a theorem of affine geometry organized as linear algebra with the linear combinations restricted to coefficients summing to 1, also called affine orbarycentric coordinates. The proof applies even to skew quadrilaterals in spaces of any dimension.

Any three points E, F, G are completed to a parallelogram (lying in the plane containing E, F, and G) by taking its fourth vertex to be $E - F + G$. In the construction of the Varignon parallelogram this is the point $(A + B)/2 - (B + C)/2 + (C + D)/2 = (A + D)/2$. But this is the point H in the figure, whence $EFGH$ forms a parallelogram.

In short, the centroid of the four points A, B, C, D is the midpoint of each of the two diagonals EG and FH of $EFGH$, showing that the midpoints coincide.

A second proof requires less mathematical aptitude. By drawing in the diagonals of the quadrilateral, we notice two triangles are created for each diagonal. And by the Midline Theorem, the segment containing two midpoints of adjacent sides is both parallel and half the respective diagonal. Therefore, the sum of the diagonals is equal to the perimeter of the quadrilateral formed. Secondly, we can use vectors 1/2 the length of each side to first determine the area of the quadrilateral, and then to find areas of the four triangles divided by each side of the inner parallelogram

6.2 rectangular components of the moment of a force:

What is a Moment? a person standing on a balcony causing a moment to occur The Moment of a force is a measure of its tendency to cause a body to rotate about a specific point or axis. This is different from the tendency for a body to move, or translate, in the direction of the force. In order for a moment to develop, the force must act upon the body in such a manner that the body would begin to twist. This occurs every time a force is applied so that it does not pass through the centroid of the body. A moment is due to a force not having an equal and opposite force directly along it's line of action.

Imagine two people pushing on a door at the doorknob from opposite sides. If both of them are pushing with an equal force then there is a state of equilibrium. If one of them would suddenly jump back from the door, the push of the other person would no longer have any opposition and the door would swing away. The person who was still pushing on the door created a moment.

The magnitude of the moment of a force acting about a point or axis is directly proportinoal to the distance of the force from the point or axis. It is defined as the product of the force (F) and the moment arm (d). The moment arm or lever arm is the perpendicular distance between the line of action of the force and the center of moments.

Moment = Force x Distance or M = (F)(d)

The Center of Moments may be the actual point about which the force causes rotation. It may also be a reference point or axis about which the force may be considered as causing rotation. It does not matter as long as a specific point is always taken as the reference point. The latter case is much more common situation in structural design problems.

A moment is expressed in units of foot-pounds, kip-feet, newton-meters, or kilonewton-meters. A moment also has a sense; A clockwise rotation about the center of moments will be considered a positive moment; while a counter-clockwise rotation about the center of moments will be considered negative.

A moment can also be considered to be the result of forces detouring from a direct line drawn between the point of loading of a system and its supports. In this case, the blue force is an eccentric force. In order for it to reach the base of the column, it must make a detour through the beam. The greater the detour, the greater the moment. The most efficient structural systems have the least amount of detours possible.

There are cases in which it is easier to calculate the moments of the componenets of a force around a certain point than it is to calculate the moment of the force itself. It could be that the determination of the perpendicular distance of the force is more difficult than determining the perpendicular distance of components of the force. The moment of several forces about a point is simply the algebraic sum of their component moments about the same point. When adding the moments of componenets, one must take great care to be consistant with the sense of each moment. It is often prudent to note the sense next to the moment when undertaking such problems.

When force is applied to an object at a certain point, it does two things: push the object, and rotate the object. The amount of that rotational tendency is described by the moment of force. A moment of force is a vector: it has both a magnitude (the strength of the rotational force) and a direction (the axis along which the rotation will take place). The direction can be determined using the right hand rule: with your thumb pointed along the moment of force, your fingers curl in the direction of rotation. Calculating the moment of force is simple vector math.

The moment of a force about a point is a measure of the tendency of the force to rotate a body about that point. The body does not necessarily have to rotate about that point, but the moment defines how the force is trying to rotate the body. A good example of a moment is when a force applied is applied to a wrench. The wrench may not actually turn, but the force generates a turning force (i.e. moment) around the bolt.

Moments are often referred to as torques, and the terms can be used interchangeably. In scalar notation, a moment about a point O is Mo = r F Here F is the magnitude of the force and r is the perpendicular distance to the line of action of the force.

The orientation of the moment is in the same direction as the rotation of the body if the body were allowed to rotate. A moment can be symbolized as a curved vector around the rotation point.

If a force is applied at the point O or the line of action of the force passes through point O, then the moment about point O is zero, and the force has no tendency to rotate the body about that point.

The principle of moments, which is also referred to as Varignon's theorem, states that the moment of a force about a point is equal to the sum of the moments of the force's components about the point. Therefore, if the components of a force are known, it may be simpler to determine the moment of each component and then add them.

Torque, moment or moment of force is the tendency of a force to rotate an object about an axis, fulcrum, or pivot. Just as a force is a push or a pull, a torque can be thought of as a twist to an object. Mathematically, torque is defined as the cross product of the lever-armdistance vector and the force vector, which tends to produce rotation.

Loosely speaking, torque is a measure of the turning force on an object such as a bolt or a flywheel. For example, pushing or pulling the handle of a wrench connected to a nut or bolt produces a torque (turning force) that loosens or tightens the nut or bolt.

The symbol for torque is typically τ, the Greek letter *tau*. When it is called moment, it is commonly denoted *M*.

The magnitude of torque depends on three quantities: the force applied, the length of the *lever arm* connecting the axis to the point of force application, and the angle between the force vector and the lever arm. In symbols:

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$\tau = \|\mathbf{r}\| \, \|\mathbf{F}\| \sin \theta$$

where

$\boldsymbol{\tau}$ is the torque vector and τ is the magnitude of the torque,
r is the displacement vector (a vector from the point from which torque is measured to the point where force is applied),
F is the force vector,
× denotes the cross product,
θ is the angle between the force vector and the lever arm vector.

The length of the lever arm is particularly important; choosing this length appropriately lies behind the operation of levers, pulleys, gears, and most other simple machines involving a mechanical advantage.

Terminology

In the UK and in US mechanical engineering, this is called moment of force, usually shortened to moment. In US mechanical engineering, the term torque means "the resultant moment of a Couple," and (unlike in UK physics), the terms torque and moment are not interchangeable.

Torque is defined mathematically as the rate of change of angular momentum of an object. The definition of torque states that one or both of the angular velocity or the moment of inertia of an object are changing. Moment is the general term used for the tendency of one or more applied forces to rotate an object about an axis, but not necessarily to change the angular momentum of the object (the concept which is called torque in physics).[5] For example, a rotational force applied to a shaft causing acceleration, such as a drill bit accelerating from rest, results in a moment called a torque. By contrast, a lateral force on a beam produces a moment (called a bending moment), but since the angular momentum of the beam is not changing, this bending moment is not called a torque. Similarly with any force couple on an object that has no change to its angular momentum, such moment is also not called a torque.

History

The concept of torque, also called moment or couple, originated with the studies of Archimedes on levers. The rotational analogues of force, mass, and acceleration are torque, moment of inertia and angular acceleration, respectively.

Definition and relation to angular momentum

A particle is located at position r relative to its axis of rotation. When a force F is applied to the particle, only the perpendicular component $F_\perp$ produces a torque. This torque $\tau = r \times F$ has magnitude $\tau = |r|\,|F_\perp| = |r|\,|F| \sin\theta$ and is directed outward from the page.

A force applied at a right angle to a lever multiplied by its distance from the lever's fulcrum (the length of the lever arm) is its torque. A force of three newtons applied two metres from the fulcrum, for example, exerts the same torque as a force of one newton applied six metres from the fulcrum. The direction of the torque can be determined by using the right hand grip rule: if the fingers of the right hand are curled from the direction of the lever arm to the direction of the force, then the thumb points in the direction of the torque.