EQUILIBRIUM OF A RIGID BODY AND ANALYSIS OF ETRUCTURAS II

9.1 reactions in supports and joints of a two-dimensional structure and statically indeterminate reactions:

- Statically indeterminate structures are the ones where the independent reaction components, and/or internal forces cannot be obtained by using the equations of equilibrium only.

To solve indeterminate systems, we must combine the concept of equilibrium with compatibility.

• Advantages. There are several advantages in designing indeterminate structures. These include the design of lighter and more rigid structures. With added redundancy in the structural system, there is an increase in the overall factor of safety.

• Several classical methods have been developed to solve for the forces and displacements of statically indeterminate systems. In this course, we will introduce two important classical approaches: the force method and the slope-deflection method.

Statically indeterminate structures are those which can be solved by using the equations of static equilibrium alone. Additional compatibility equations are required to solve indeterminate structures e.g. fixed ended beam, continuous beam, propped cantilever etc.

It is very important to understand the behavior of different types of supports. In general every hinged support will have two reaction components (along x and y axes). Roller support will have one reaction in the direction perpendicular to the surface on which the rollers are kept. Fixed support will have three reactions; two in x and y directions and the third is moment about z-axis at the support.

If the number of unknown reaction components are equal to the number of equations, the structure is known as **statically determinate**.

If the unknowns are more than the number of equations of equilibrium the structure is known as statically indeterminate. The difference between known reaction components and the equations of equilibrium is known as degree of indeterminacy.

Therefore the beam is statically indeterminate of degree one as it has 4 reactions and only 3 equations of equilibrium.

It the unknown reaction components are less than the number of equilibrium equation, the structure is known as unstable. A structure is also defined as unstable if all the reaction components are concurrent or parallel (even if reactions are equal or more than the numbers of equations.) this is evident from the beam that if we change the hinged support at C to a roller support, all the reactions will be vertical and hence parallel to each other and this beam will not be able to support any horizontal force applied to it.

Most of the structures being used in construction are indeterminate in nature. Force method of analysis or displacement method of analysis could be used to solve for It is generally classified into two categories as Determinate and Indeterminate structures or Redundant Structures indeterminate structures. Some of the common methods to solve for indeterminate structures are: stiffness method, moment distribution method, flexibility method, energy theorems, slope deflection equations, etc.

Force Method –Basic Idea.
The basic idea of this method is to identify the redundant forces first. Then using the compatibility conditions, determine the redundant forces. This way the structure is essentially reduced to a statically determinate structure. All the remaining forces can be determined using the equations of equilibrium

Structure is an assemblage of a number of components like slabs, beams, columns, walls, foundations and so on, which remains in equilibrium. It has to satisfy the fundamental criteria of strength, stiffness, economy, durability and compatibility, for its existence..

Any structure is designed for the stress resultants of bending moment, shear force, deflection, torsional stresses, and axial stresses. If these moments, shears and stresses are evaluated at various critical sections, then based on these, the proportioning can be done. Evaluation of these stresses, moments and forces and plotting them for that structural component is known as analysis. Determination of dimensions for these components of these stresses and proportioning is known as design.

Determinate structures are analysed just by the use of basic equilibrium equations. By this analysis, the unknown reactions are found for the further determination of stresses. Redundant or indeterminate structures are not capable of being analysed by mere use of basic equilibrium equations. Along with the basic equilibrium equations, some extra conditions are required to be used like compatibility conditions of deformations etc to get the unknown reactions for drawing bending moment and shear force diagrams.

Example of determinate structures are: simply supported beams, cantilever beams, single and double overhanging beams, three hinged arches, etc.

Examples of indeterminate structures are: fixed beams, continuous beams, fixed arches, two hinged arches, portals, multistoried frames, etc.

Special methods like strain energy method, slope deflection method, moment distribution method, column analogy method, virtual work method, matrix methods, etc are used for the analysis of redundant structures.

Loads on a beam are applied in a plane containing an axis of symmetry
Beams have one or more points of support referred to as reactions but in this course they will be more often referred to as nodes. Nodes A, B, and C represent reactions. Node D identifies a location on the beam (the free end) where we wish to extract information. Beams deflect in the plane of the loads. Internal forces consist of shear forces, bending moments, torques (take CVE 513), and axial loads Shear Moment Axial load V M A Actions y θ x Displacements (translations, rotations)

All structural components are in same plane.
Forces act in the plane of structure.
External forces and reactions to those forces are considered to act only at the nodes and result in forces in the members which are either tensile or compressive forces. Thus all members are two force members.

Loads acting on members are replaced by statically equivalent forces at the joints. So the moment M1, the distributed load w and the force P4 would have to be replaced by equivalent joint loads to conduct an analysis.

Joints are assume hinged, so no bending moments are transmitted through a joint and absolutely no twisting moments can be applied to the truss (consider a gusset plate).

Forces and structural elements are no longer confined to a plane. A space frame truss is a three-dimensional framework of members pinned at their ends. A tetrahedron shape is the simplest space truss, consisting of six members which meet at four joints. Large planar structures may be composed from tetrahedrons with common edges. Space trusses are employed in the base structures of large freestanding power line pylons.

As in planar trusses only axial tensile or compressive forces can be developed. Elements can intersect at rigid or flexible connections All forces are normal to the plane of the structure. Typically used to support roofs with no internal column support (think of indoor sports arenas).

All couples have their vectors in the plane of the grid. Torques can be sustained. Each member is assumed to have two axes of symmetry so that bending and torsion can occur independently of one another (see unsymmetrical bending in CVE 513).

Joints are no longer required to be hinges. They can be rigid, or they can sustain rotation. Forces and deflection are contained in the plane X-Y. All couples have moment vectors parallel to Z-axis. Internal resultants consist of bending moments, shearing forces and axial forces. Joints may transfer moment Most general type of framed structure. No restrictions on location of joints, directions of members, or directions of loads. Members are assumed to have two axes of symmetry for the same reason grids have two axes of symmetry.

The continuity of the displacements throughout the structure must be satisfied in a correct structural analysis. This is sometimes referred to as conditions of geometry. As an example, compatibility conditions must be satisfied at all points of support. If a horizontal roller support is present then the vertical displacement must be zero at that support.

We always impose compatibility at a joint. If two structural elements frame into a joint then there displacements and rotations at the connection must be the same or consistent with each other. We apply a much more rigorous mathematical definition in CVE 604 for compatibility. It is simply noted here that strain is a function of displacement. There are 6 components of strain and only 3 components of displacement at a point in a three dimensional analysis. A “compatible” displacement field will produce an appropriate state of strain at a point.

Flexibility methods use equations that express the compatibility of the displacements. Understanding this issue as it applies to structural analyses give the student a better “feel” as to how a structure behaves and an ability to judge the correctness of a solution.

General procedure for beams with a single redundant.

Step 1: Identify the redundant. If the redundant is removed from the original structure, the resulting beam must be stable and determinate. Now create the two beams whose superposition results in the original indeterminate beam. –

- Remove the redundant from the original beam but leave the external loads. This is beam DSRL (Determinate Structure with Real Loads) – beam I.

- Remove the redundant and all loads from the original beam. Assume a direction for the redundant. Now apply a unit force or moment along the assumed direction of the redundant. This is beam DSUL (Determinate Structure with Unit Load) – beam II. –

- Write the single compatibility equation in the symbolic form. Select a sign convention for the associated displacements appearing in the equation. This equation should contain the redundant.

Step 2: Compute the deflection for the beam DSRL.
Step 3: Compute the deflection for the beam DSUL.
Step 4: Substitute the deflections from Steps 2 and 3 into the compatibility equation. Use the sign convention to assign the correct sign to the two displacements. Solve the compatibility equation for the redundant. If the answer is positive, the assumed direction for the redundant is correct. Otherwise, flip the direction.

Step 5: The other support reactions can now be computed using the free-body diagram of the original beam (or through superposition of the two determinate beams).

General procedures for internally indeterminate trusses

Step 1: Identify the redundant member (ij). If the member is removed from the original structure, the resulting truss must be stable and determinate. Now create the two trusses whose superposition results in the original indeterminate truss.

• Remove the redundant from the original truss but leave the external loads. This is truss DTRL.
• Remove the redundant and all loads from the original truss. Assume that the redundant member is in tension. Now apply unit tensile forces along the redundant member. This is truss DTUL.
• Write the single compatibility equation in the symbolic form. This equation should contain the redundant member force Fij

Step 2: Compute the displacement along ij for the truss DTRL.
Step 3: Compute the displacement along ij for the truss DTUL.
Step 4: Now substitute the displacement from Steps 2 and 3 into the compatibility equation. Solve the compatibility equation for the redundant. If the answer is positive, the redundant is in tension. Otherwise, the member is in compression.
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Step5: The other member forces can be computed through superposition of the two determinate trusses.