STATISTICS

THE EQUATIONS

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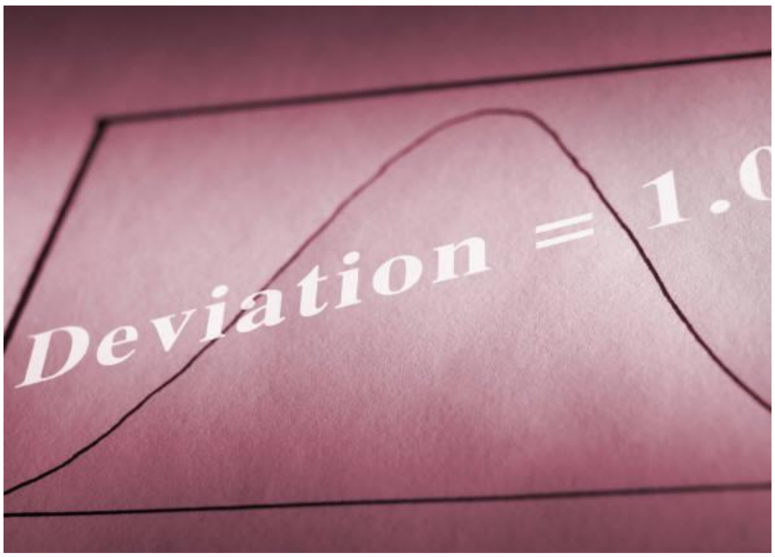
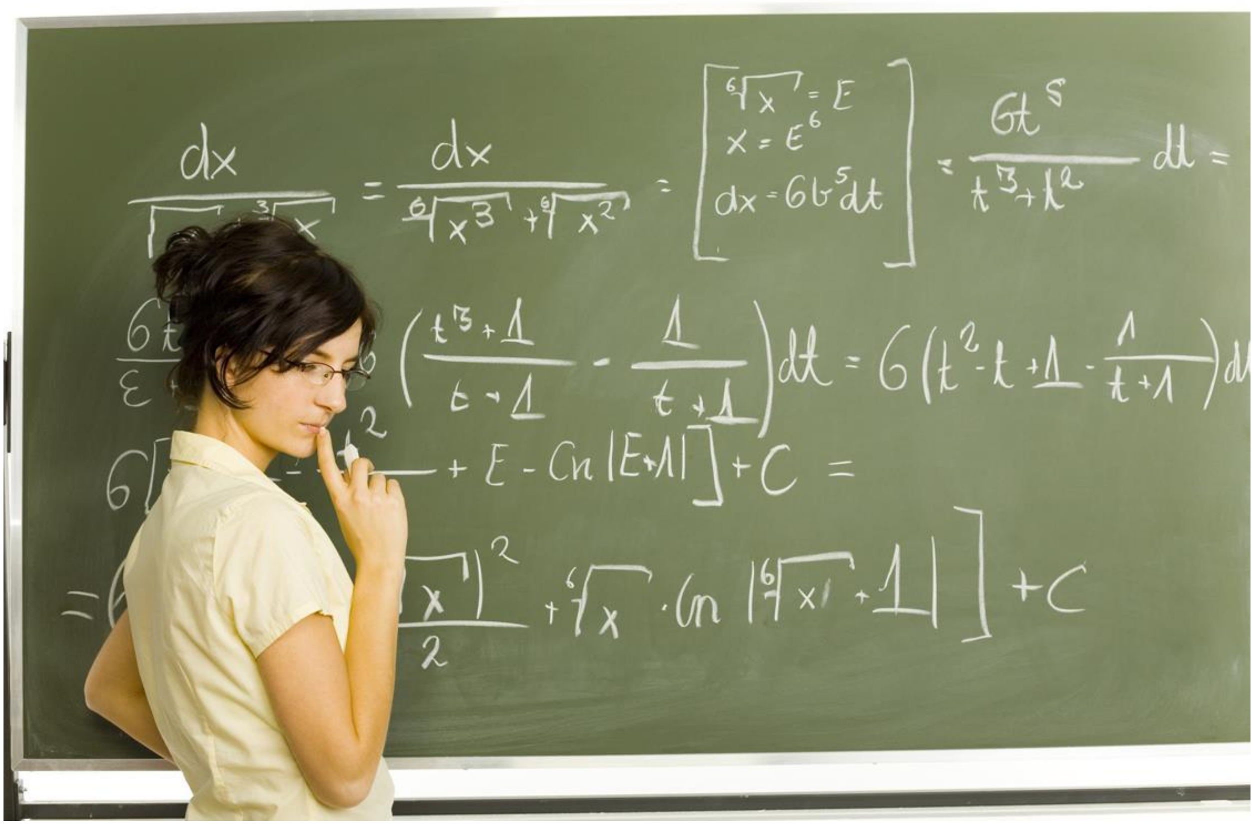
SESSION 2 EQUATIONS

A First Lesson in Algebra:





Solving Equations!

One of the first lessons taught in Algebra is Solving Equations. This is the basis of Algebra and many other lessons taught in Algebra will rely on knowledge of this skill.

So, what is an equation?

An equation is a mathematical statement that shows that two quantities are equal.

The Mean and Median: Measures of Central Tendency

The **mean** and the **median** are summary measures used to describe the most "typical" value in a set of values.

Statisticians refer to the mean and median as **measures of central tendency**.

The Mean and the Median

The difference between the mean and median can be illustrated with an example. Suppose we draw a sample of five women and measure their weights. They weigh 100 pounds, 100 pounds, 130 pounds, 140 pounds, and 150 pounds.

- To find the **median**, we arrange the observations in order from smallest to largest value. If there is an odd number of observations, the median is the middle value. If there is an even number of observations, the median is the average of the two middle values. Thus, in the sample of five women, the median value would be 130 pounds; since 130 pounds is the middle weight.

- The **mean** of a sample or a population is computed by adding all of the observations and dividing by the number of observations. Returning to the example of the five women, the mean weight would equal (100 + 100 + 130 + 140 + 150)/5 = 620/5 = 124 pounds. In the general case, the mean can be calculated, using one of the following equations:

  Population mean = $\mu = \Sigma X / N$ OR Sample mean = $x = \Sigma x / n$

  where ΣX is the sum of all the population observations, N is the number of population observations, Σx is the sum of all the sample observations, and n is the number of sample observations.

When statisticians talk about the mean of a population, they use the Greek letter μ to refer to the mean score. When they talk about the mean of a sample, statisticians use the symbol x to refer to the mean score.

The Mean vs. the Median

As measures of central tendency, the mean and the median each have advantages and disadvantages. Some pros and cons of each measure are summarized below.

- The median may be a better indicator of the most typical value if a set of scores has an **outlier**. An outlier is an extreme value that differs greatly from other values.

- However, when the sample size is large and does not include outliers, the mean score usually provides a better measure of central tendency.

To illustrate these points, consider the following example. Suppose we examine a sample of 10 households to estimate the typical family income. Nine of the households have incomes between $20,000 and $100,000; but the tenth household has an annual income of $1,000,000,000. That tenth household is an outlier. If we choose a measure to estimate the income of a typical household, the mean will greatly over-estimate the income of a typical family (because of the outlier); while the median will not.

Effect of Changing Units

Sometimes, researchers change units (minutes to hours, feet to meters, etc.). Here is how measures of central tendency are affected when we change units.

- If you add a constant to every value, the mean and median increase by the same constant. For example, suppose you have a set of scores with a mean equal to 5 and a median equal to 6. If you add 10 to every score, the new mean will be 5 + 10 = 15; and the new median will be 6 + 10 = 16.
- Suppose you multiply every value by a constant. Then, the mean and the median will also be multiplied by that constant. For example, assume that a set of scores has a mean of 5 and a median of 6. If you multiply each of these scores by 10, the new mean will be 5 * 10 = 50; and the new median will be 6 * 10 = 60.

Test Your Understanding of This Lesson

Problem 1

Four friends take an IQ test. Their scores are 96, 100, 106, 114. Which of the following statements is true?

I. The mean is 103.
II. The mean is 104.
III. The median is 100.
IV. The median is 106.

(A) I only
(B) II only
(C) III only
(D) IV only
(E) None is true

Solution

The correct answer is (B). The mean score is computed from the equation:

$$\text{Mean score} = \Sigma x / n = (96 + 100 + 106 + 114) / 4 = 104$$

Since there are an even number of scores (4 scores), the median is the average of the two middle scores. Thus, the median is (100 + 106) / 2 = 103.

Sets and Subsets

The lesson introduces the important topic of sets, a simple idea that recurs throughout the study of probability and statistics.

Set Definitions

- A **set** is a well-defined collection of objects.
- Each object in a set is called an **element** of the set.
- Two sets are **equal** if they have exactly the same elements in them.
- A set that contains no elements is called a **null set** or an **empty** set.
- If every element in Set *A* is also in Set *B*, then Set *A* is a **subset** of Set *B*.

Set Notation

- A set is usually denoted by a capital letter, such as *A, B,* or *C*.
- An element of a set is usually denoted by a small letter, such as *x, y,* or *z*.
- A set may be described by listing all of its elements enclosed in braces. For example, if Set *A* consists of the numbers 2, 4, 6, and 8, we may say: *A* = {2, 4, 6, 8}.
- The null set is denoted by {∅}.
- Sets may also be described by stating a rule. We could describe Set *A* from the previous example by stating: Set *A* consists of all the

For example, 6 + 3 = 9. This is an equation that shows that the expression 6+3 is equal to the quantity 9.

However, in Algebra, one of the terms is typically unknown and a variable (letter) is used in it's place. For example, $x + 3 = 9$ is an Algebra equation. We must solve this equation to determine the value of x.

before we get into the meat of our Solving Equations unit, we are going to quickly take a look at balancing equations.

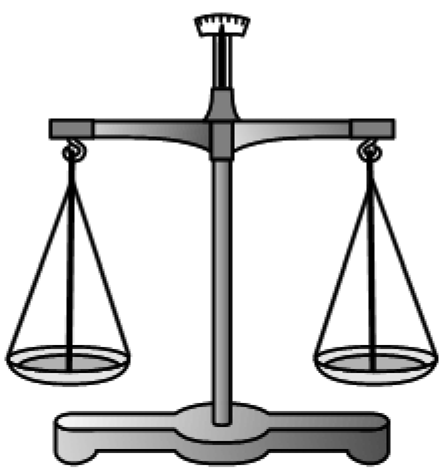
As you begin solving equations in Algebra, the very first thing that you will learn is that whatever you do, you MUST keep your equation balanced!

It truly is the number one factor in solving equations!

You will see step-by-step how equations are kept balanced for every example in this unit! In the explanations, you will see the words:

"Whatever you do to one side, you must do to the other side!"

Those are the key words for balancing equations! Let's take a look at a graphic to give you a better understanding.

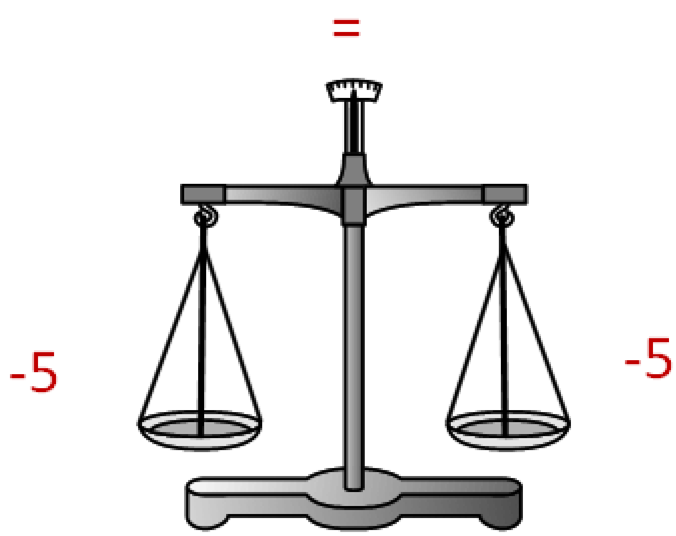


Think of a scale that you would use to weigh things! If you put a 5 lb weight on the left side, what happens? Is the scale balanced? NO... of course not! What would you have to do to balance it perfectly? You'd have to put another five pound weight (or something that weighs five pounds) on the other side.

So... if I put 5 lbs one side, then I MUST also put 5 lbs on the OTHER side! That's the only way it will stay balanced!

So... does this phrase make sense...?

"Whatever you do to one side, you must do to the other side!"



Think of the equation sign as the "center" of the scale.

Take a look at the picture above, and imagine two 5 lb weights on the left side, and two 5 lb weights on the right side. How much weight is on each side? 10 lbs, right? So, right now the scale is balanced!

Now, what happens if I take 5 lbs off of the right side? Is the scale balanced now? No, there's 10 lbs still on the left, but only 5 lbs on the right.

So, what must I do to keep it balanced? Yes.... I must ALSO take 5 lbs off of the left! Now, it is balanced again!

.

So, in Algebra if you subtract 5 from one side, you must also subtract 5 from the other side.

Again....

"Whatever you do to one side, you must do to the other side!"

Why have I continued to repeat those same words?

Because... those are the words that you MUST remember with EVERY step of EVERY equation that you solve!

You will see those words in all of my explanations, so look for them!

Solving One-Step Equations

Multiplication Equations

Solving one-step equations involving multiplication are easy as long as you can divide!

The next set of one-step equations do not contain a constant that you must add or subtract to remove.

These equations contain a coefficient. A coefficient is a number that is multiplied by the variable.

Therefore, we must remove the coefficient. Take a look at this equation: $3x = 9$. Since there is no mathematical symbol between the 3 and the x we know that means multiplication. So, what number times 3 will give us an answer of 9?

You know the answer, right? Yes, 3! $3*3 = 9$

Another question to ponder- What is the opposite of multiplication? Yes... division! We are going to divide in order to get x by itself!

Why divide? What is 3/3? Yes... 1! What is 1*x? You got it.... x! That's how we get x by itself.

We want the coefficient to be 1. Anytime you divide a number by itself, you will get an answer of 1!

Let's look at a few examples:

Example 1

solving multiplication equations

Pretty easy, huh? I think multiplication equations are even easier than addition and subtraction equations. Keep working, you'll get the hang of it!

Example 2

solving one-step equations

Using the Distributive Property when Solving Equations

Now is your chance to learn how to use the distributive property and combining like terms in order to solve more complex equations.

It seems pretty easy to learn all of these skills in isolation, but using them together to solve one problem is the key in Algebra 1.

So, how does this work? Here are a few steps to take when you come across an algebra equation that looks a little more challenging.

Steps for Solving Algebra Equations

- If you see parenthesis, with more than one term inside, the distribute first!
- Rewrite your equations with like terms together. Take the sign in front of each term.
- Combine like terms.
- Continue solving the one or two-step equation.

Let's look at a couple of examples to clarify those steps for you.

This first example is a pretty basic equation that involves the distributive property. Take note of how I distribute first before applying the rules for solving equations.

Example 1

$3(x-2) = 21$	Original Problem – You have a set of parenthesis with 2 terms inside - This means distribute!
$3x - 6 = 21$	Distribute! 3•x - 3•(2). Now you have a two-step equation!
$3x - 6 + 6 = 21 + 6$	Add 6 to BOTH sides.
$3x = 27$	Now you have a one-step equation.
$\frac{3x}{3} = \frac{27}{3}$	Divide by 3 on BOTH sides.
$x = 9$	Solution of 9.
Check! $3(x-2) = 21$ $3(9-2) = 21$ $3(7) = 21$ $21 = 21$ ☺	Check your solution by substituting back into the original equation. Make sure both sides are equal.

That example was pretty easy, I know! Let's look at one that requires a few more steps.

This next example looks more confusing because the distributive property comes right in the middle of the equation. You still must distribute first and then combine like terms before solving the equation.

Example 2

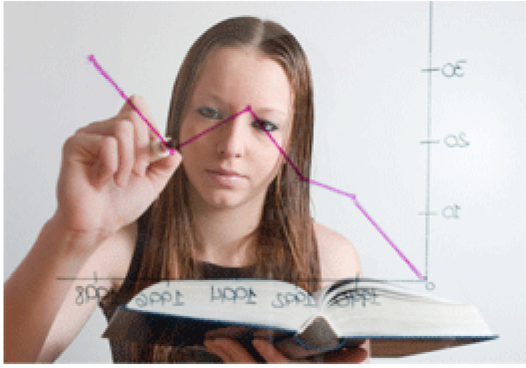
$2x - 3(2x - 3) + 4 = 33$	Original Problem – You have a set of parenthesis with 2 terms inside - This means distribute!
$2x - 6x + 9 + 4 = 33$	Distribute! $-3 \cdot 2x = -6x$ & $-3 \cdot -3 = 9$
$-4x + 13 = 33$	Like terms were already written together – so we just needed to combine like terms!
$-4x + 13 - 13 = 33 - 13$	Solve your two-step equation. Subtract 13 from BOTH sides.
$-4x = 20$	Now you have a one-step equation.
$\frac{-4x}{-4} = \frac{20}{-4}$	Divide by -4 on BOTH sides.
$x = -5$	Solution of -5
Check $2x - 3(2x - 3) + 4 = 33$ $2 \cdot (-5) - 3(2 \cdot -5 - 3) + 4 = 33$ $33 = 33$ ☺	Check your solution by substituting back into the original equation. Make sure both sides are equal.

Ok... just one more example. This one is a little more difficult. You will have to distribute twice and you must make sure that when you distribute the 4, you actually distribute a negative 4.

Remeber to take the sign in front. Pay close attention to the signs in this example.

Example 3

$3(2x + 2) - 4(x - 5) = 18$	Original Problem – You have 2 sets of parenthesis with 2 terms inside - This means distribute twice!
$6x + 6 - 4x + 20 = 18$	Distribute both sets of parenthesis.
$6x - 4x + 6 + 20 = 18$	Rewrite the problem with like terms together. Take the sign in front of the term with it!
$2x + 26 = 18$	Combine like terms. Now you have a two-step equation.
$2x + 26 - 26 = 18 - 26$	Subtract 26 from BOTH sides.
$2x = -8$	Now you have a one-step equation.
$\frac{2x}{2} = \frac{-8}{2}$	Divide by 2 on BOTH sides.
$x = -4$	Solution: $x = -4$
Check: $3(2x + 2) - 4(x - 5) = 18$ $3(2 \cdot -4 + 2) - 4(-4 - 5) = 18$ $18 = 18$ ☺	Check your solution by substituting back into the original equation. Make sure both sides are equal.



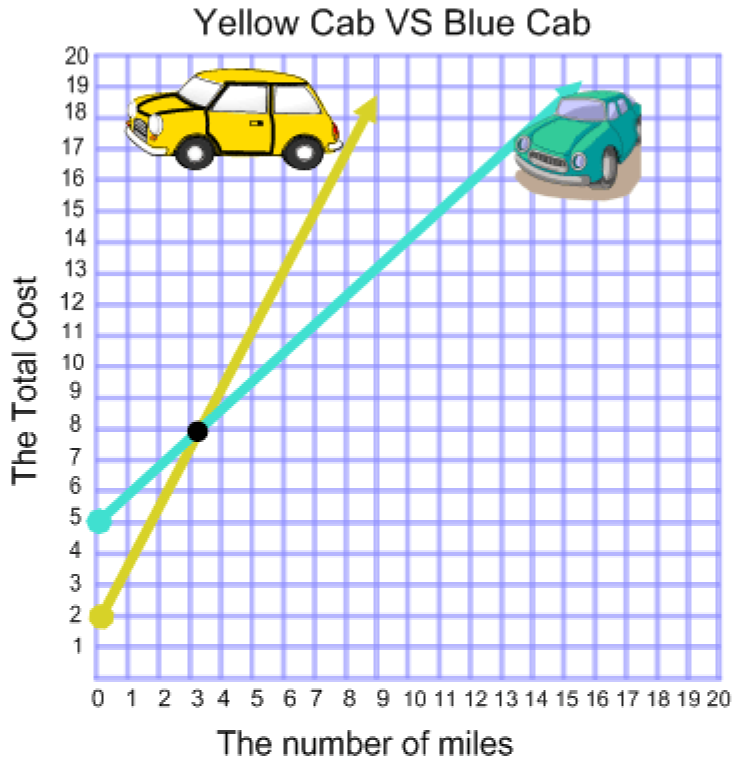
Graphing equations is the heart of Algebra! Especially graphing linear equations, which will be the focus of this unit. You'll find that when working with those impossible word problems, a graph can give you an unbelievable amount of information and help you to solve the problem more easily. You'll also find that linear graphs are used in many real world situations.

So, get your graph paper, ruler, and pencil ready and let's get started graphing.

System of Equations

So, what is a system of equations? This may be a new term for you if you are just beginning your study of Algebra.

A system of equations is a set of two or **more** equations that you deal with at one time. When solving the system, you must consider all of the equations involved and find a solution that satisfies all of the equations. Dealing with more than one equation is what intimidates some students, but it's really not that hard.



This graph is an example of a System of Equations. These two linear graphs represent the cost of taking a cab around town based on the number of miles driven.

Since we are working with a system, we must graph both of the equations on the same graph.

When you graph a system, the **point of intersection** is the solution. The point of intersection on this graph is (3,8). That means that both companies will charge the same amount, $8 for 3 miles.

That is the only time that the two companies will charge the same amount. Therefore, the point (3,8) is the solution.

A **linear system** of equations will only have **one solution,** and that is the point of intersection. Although, as always, there are times when you will **find** no solution or an infinite number of solutions, and we will cover those special situations in the lessons below.

- Types of equations

Equations can be classified according to the types of operations and quantities involved. Important types include:

- An algebraic equation or polynomial equation is an equation in which both sides are polynomials (see also system of polynomial equations). These are further classified by degree:
    - linear equation for degree one
    - quadratic equation for degree two
    - cubic equation for degree three
    - quartic equation for degree four
    - quintic equation for degree five
- A Diophantine equation is an equation where the unknowns are required to be integers
- A transcendental equation is an equation involving a transcendental function of its unknowns
- A parametric equation is an equation for which the solutions are sought as functions of some other variables, called parameters appearing in the equations
- A functional equation is an equation in which the unknowns are functions rather than simple quantities
- A differential equation is a functional equation involving derivatives of the unknown functions
- An integral equation is a functional equation involving the antiderivatives of the unknown functions
- An integro-differential equation is a functional equation involving both the derivatives and the antiderivatives of the unknown functions
- A difference equation is an equation where the unknown is a function f which occurs in the equation through $f(x)$, $f(x-1)$, ..., $f(x-k)$, for some whole integer k called the *order* of the equation. If x is restricted to be an integer, a difference equation is the same as a recurrence relation