Chapter 8 The Costs of Production

Chapter 8:

The Costs of Production

So far we have looked at one aspect of the production process — resource productivity. We will now examine the other crucial component — costs. Ultimately, costs help determine the resource mix a firm will use, how much output a firm will produce, whether profit is realized, and whether a firm will continue to produce in the long run.

Costs in the Short Run

Production costs are broken down into two broad categories: fixed costs and variable costs. Total costs are the sum of all fixed and variable costs and can be expressed as:

$$TC = TFC + TVC$$

where *TC* is total costs, *TFC* is total fixed costs, and *TVC* is total variable costs.

Fixed Costs

Fixed costs arise because some inputs are fixed in the short run. For example, plant size and capital are typically fixed in the short run, and payments for their use — monthly rent, property taxes, loan payments for capital, etc., — are costs a firm incurs regardless of the level of production: 1,000 units a day, 100 units a day, or 0 units a day.

Fixed costs are the sum of all short run costs that are unrelated to the level of output.

Managers often refer to fixed costs as *overhead*, indicating that these costs are unaffected by output.

Decision-making should not be influenced by fixed costs–such costs are *sunk*. For example, President Johnson was supposedly reluctant to bring an end to the Vietnam War in 1967 because he felt that doing so without achieving some sort of victory would mean that all the lives lost up to that point were lost in vain. Deciding whether or not to end a war, however, should not be based on sunk costs (lost lives), because sunk costs are irretrievable no matter what happens. Similarly, rational business decisions will not be determined by unrecoverable fixed costs.

Fixed costs are meaningful only to the extent that, like history or archeology, we can learn from them. Since they are fixed, there is a sense in which no alternative exists, so the opportunity costs of fixed resources are zero in the short run. Therefore, only opportunity costs should affect production decisions — costs that vary with output because alternatives are foregone while incurring these costs.

Variable Costs

Variable costs are incurred when labor, raw materials, or other variable inputs are used.

Variable costs depend on the level of production and are incurred when output is produced.

Table 8-1 and Figure 8-1 give monthly cost data for your latest venture: Radical Rollerblades. To keep things simple, labor is your only variable input. Each worker is paid $2,000 a month, so your variable costs equal the wage rate (w) of $2,000 multiplied by the number of workers (L) you hire ($TVC = w \times L$).

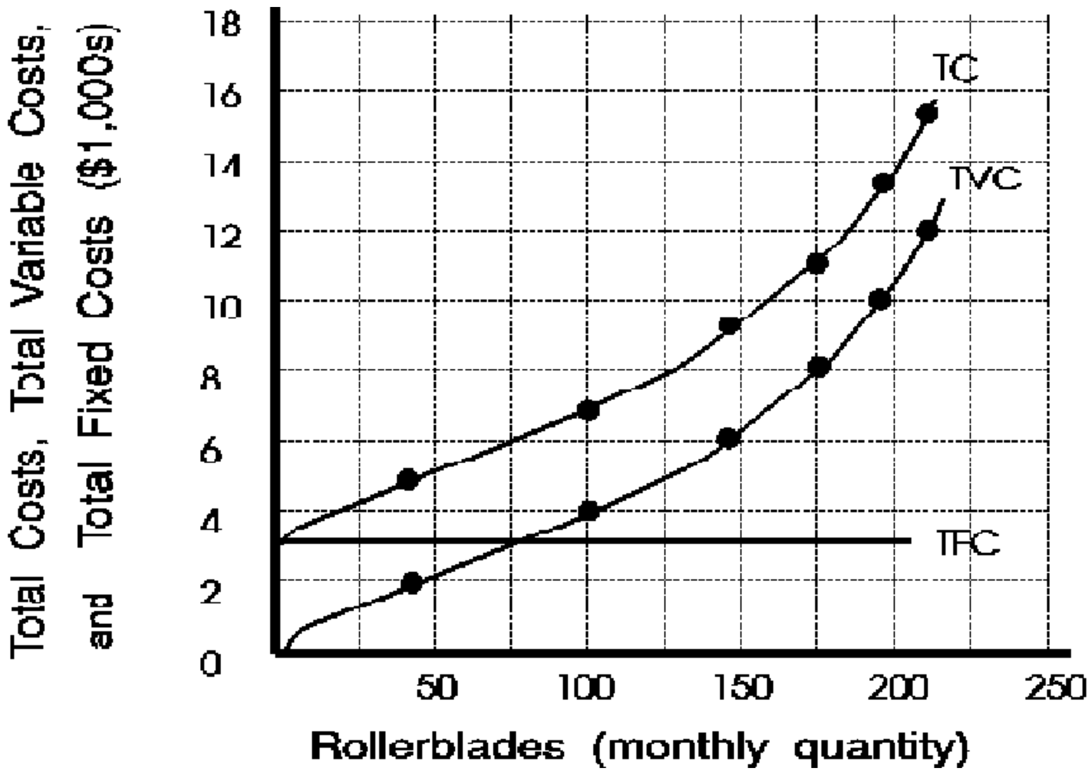
Table 8-1 Output and Total Costs at Radical Rollerblades

Labor (L)	Output (Q)	Total Fixed Costs (TFC)	Total Variable Costs (TVC)	Total Costs (TC)
0	0	$3000	0	$3000
1	40	$3000	$2000	$5000
2	100	$3000	$4000	$7000
3	145	$3000	$6000	$9000
4	175	$3000	$8000	$11000
5	195	$3000	$10000	$13000
6	210	$3000	$12000	$15000

The shape of the *TVC* curve in Figure 8-1 reflects diminishing marginal returns. Initially, the *TVC* curve increases at a decreasing rate (the range of output from 0 to 70 units), but then the *TVC* curve increases at an increasing rate (all production beyond 70 units). When production exceeds 70 rollerblades a month, marginal returns diminish as additional labor works with a fixed amount of capital. Consequently, additional labor produces successively less output, so the *TVC* curve increases at an ever-increasing rate.

The *TC* curve is identical to the *TVC* curve in Figure 8-1, except that it is $3,000 higher at each output level. Because $TC = TVC + TFC$, this $3,000 height differential is explained by *TFC*. In Table 8-1, we see that your monthly *TFC* are indeed $3,000. Because *TFC* are constant (unaffected by the level of output), the difference between the *TC* and *TVC* curves at any level of production yields *TFC*.

Figure 8-1 Total Costs, Total Variable Costs, and Total Fixed Costs at Radical Rollerblades



The total variable cost (*TVC*) curve initially rises at a decreasing rate, but then begins to rise at an increasing rate because of diminishing marginal returns. The total fixed cost (TFC) curve is horizontal because *TFC* are incurred independently of output and are therefore constant. Since *TC* = *TVC* + *TFC*, the total cost (*TC*) curve is parallel to the *TVC* curve and lies above the *TVC* curve by a distance equal to *TFC*.

Average Costs

Costs can also be broken down into *per unit* or *average costs*. Dividing costs by output permits easy calculation of average fixed costs (*AFC*), average variable costs (*AVC*) and average total costs (*ATC*).

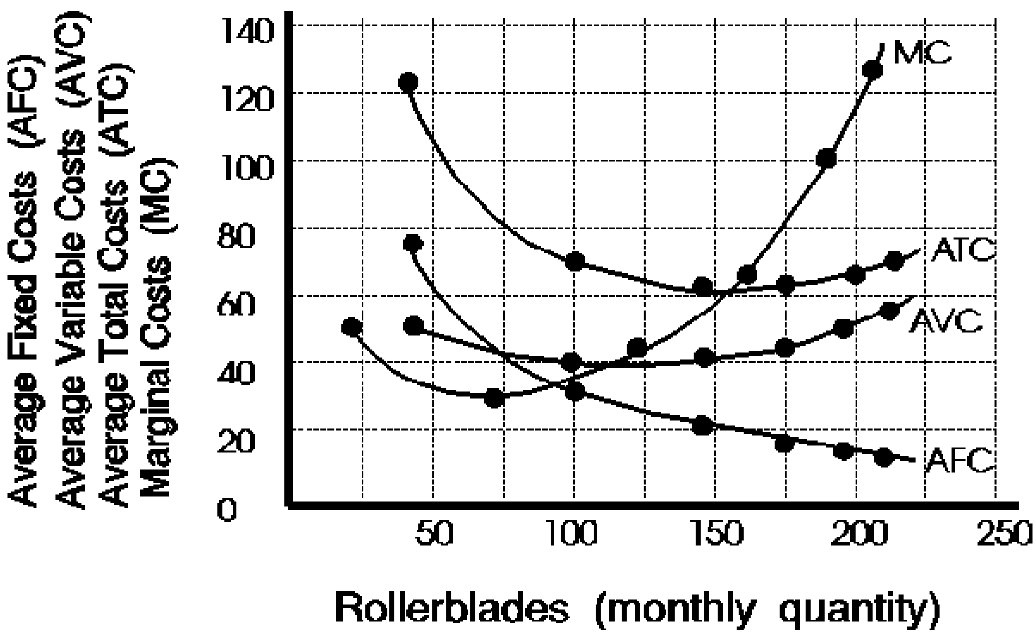
Average fixed costs (AFC) are fixed costs per unit of output, and are calculated as TFC/Q.

Looking at Table 8-2, you can see that *AFC* at Radical Rollerblades equal $75 ($3,000/40) when monthly output is 40, $30 ($3,000/100) when monthly output is 100, and so on. As output is expanded, average fixed costs decline continually because constant *TFC* are divided by greater and greater quantities of output — something known to businesspeople as "spreading overhead". Your declining *AFC* curve at Radical Rollerblades is shown in Figure 8-2.

Table 8-1 Output and Total Costs at Radical Rollerblades

Labor (L)	Output (Q)	Total Fixed Costs (TFC)	Total Variable Costs (TVC)	Total Costs (TC)	Average Fixed Costs (AFC)	Average Variable (AVC)	Average Total (ATC)	Marginal Costs (MC)
0	0	$3000	0	$3000	—	—	—	—
								$50.00
1	40	$3000	$2000	$5000	$75.00	$50.00	$125.00	
								$ 33.33
2	100	$3000	$4000	$7000	$30.00	$40.00	$ 70.00	
								$ 44.44
3	145	$3000	$6000	$9000	$20.69	$41.38	$ 62.07	
								$ 66.67
4	175	$3000	$8000	$11000	$17.14	$45.71	$ 62.85	
								$100.00
5	195	$3000	$10000	$13000	$15.38	$51.28	$ 66.67	
								$133.33
6	210	$3000	$12000	$15000	$14.29	$57.14	$ 71.43	

Figure 8-2 Average Fixed, Average Variable, Average Total, and Marginal Costs at Radical Rollerblades



The AVC, ATC, and MC curves are all U-shaped because of diminishing marginal returns. At first all three curves fall as labor productivity initially rises, but diminishing marginal returns set in, and all three cost curves begin to rise as less output is produced by each additional worker. The MC curve intersects the AVC and ATC curves at their minimum points, and "pulls" both curves down when MC is below them, and "pulls" both curves up when MC is above them. Average fixed costs (AFC) continually decline because constant TFC are "spread out" across increasing amounts of output.

Average variable costs (AVC) are variable costs per unit of output, and are calculated as TVC/Q.

Average variable costs in Table 8-2 equal $50 ($2,000/40) when 40 rollerblades are produced a month, $40 ($4,000/100) when 100 rollerblades are produced a month, and so on. The U shape of the *AVC* curve in Figure 8-2 is explained by the ever pervasive law of diminishing marginal returns. Average variable costs initially fall, but once diminishing marginal returns set in, labor productivity declines, so variable costs per unit of output (*AVC*) begin to rise.

Average total costs (ATC) are total costs per unit of output, and are calculated as TC/Q or AFC + AVC.

Table 8-2 shows that average total costs equal $125 ($5,000/40 or $75 + $50) when 40 rollerblades are manufactured a month, $70 ($7,000/100 or $30 + $40) when 100 rollerblades are manufactured a month, and so on. The average total cost curve shown in Figure 8-2 is U-shaped for the same reason the *AVC* curve is — diminishing marginal returns. Notice that as output increases, differences between the *AVC* and *ATC* curves shrink. The *ATC* and *AVC* curves converge, because their vertical differences equal *AFC*, which falls as output rises.

Marginal Cost

All rational economic decisions are made at the margin so, not too surprisingly, we are interested in the marginal cost of production.

Marginal cost (MC) is the change in total cost associated with producing an additional unit of output, and is calculated as ΔTC/ΔQ or ΔTVC/ΔQ.

Marginal costs for Radical Rollerblades are listed in Table 8-2. These values can be calculated using either *TC* or *TVC* data because the change (Δ) in *TC* equals the change in *TVC* since *TFC* is constant and independent of the level of output. For example, when production rises from 100 to 145 units monthly, *TC* rises by $2,000 ($9,000 - $7,000) as does *TVC* ($6,000 - $4,000 = $2,000). Thus, marginal cost of this extra production is $44.44 (($2,000/(145 - 100)) no matter which value you use (*TC* or *TVC*).

The marginal cost curve is U-shaped (diminishing marginal returns rears its ugly head) and intersects the *AVC* and *ATC* curves at their minima, as shown in Figure 8-2. Quick rules of thumb govern the relationship between *MC*, *ATC*, and *AVC*:

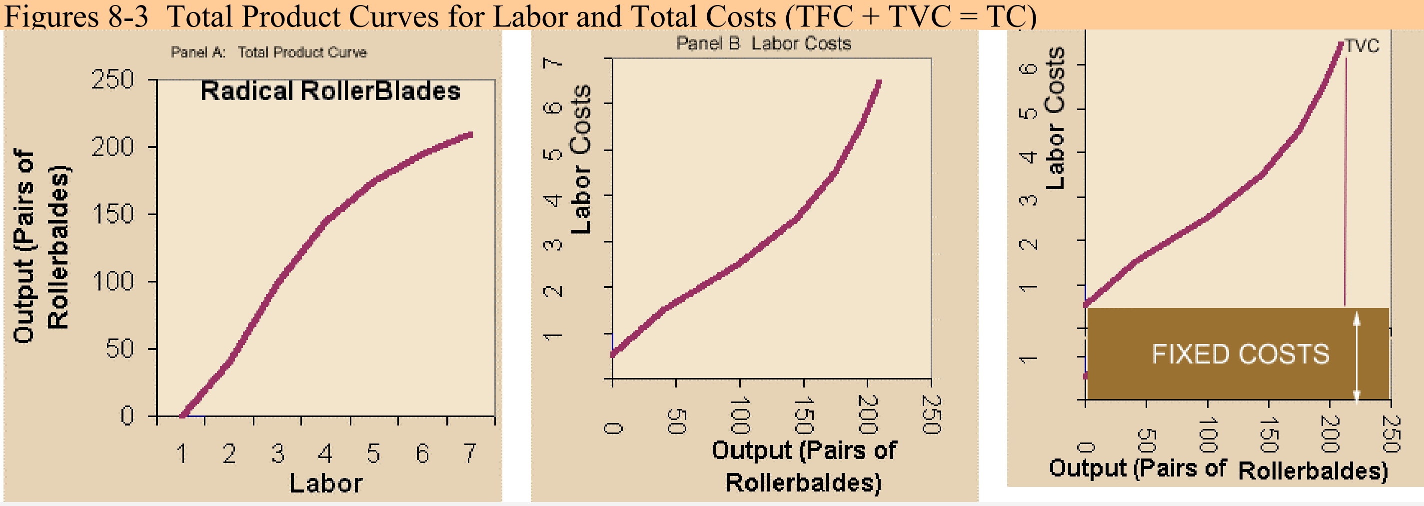
Whenever:	Then:
MC < AVC or ATC	AVC or ATC must be falling.
MC = AVC or ATC	AVC or ATC are at their minima.
MC > AVC or ATC	AVC or ATC must be rising.

The Symmetry Between Production and Costs

Short Run Total Product and Total Cost Curves The firm cannot vary at least one resource in the short run. This implies that all costs for such fixed resources are fixed. For the moment, we will assume that labor is the only variable resource for our roller blade manufacturer, so that wages are the only variable costs of production.

Panel A in Figure 8-3 depicts a Total Product curve for Radical Rollerblades. You crudely rotate this figure to generate Panel B, which shows how labor costs rise as output rises, by drawing a typical total product curve on a clean sheet of paper, turning it backwards, and then holding the page upwards and aiming it a good light source. A bit of pivoting should prove our point for this exercise, which is that the total product curve is very tightly related to the total variable cost curve.

Figures 8-3 Total Product Curves for Labor and Total Costs (TFC + TVC = TC)

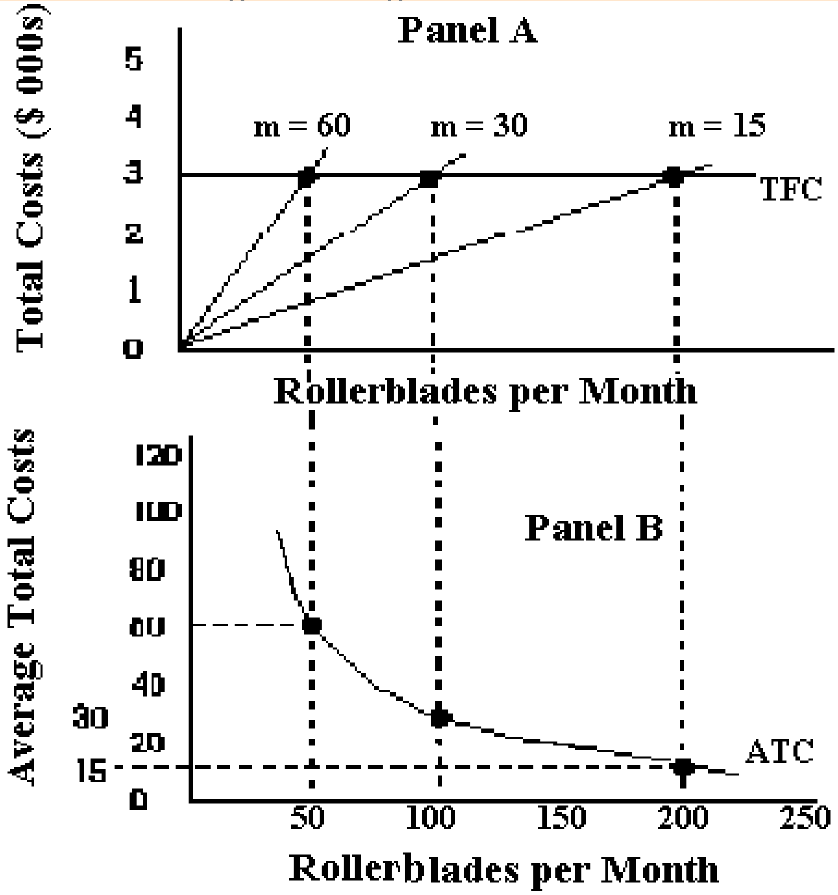


Let's try one further exercise to reinforce that a tight relationship exists between production functions and cost functions. Take the page you've just used to draw a total product curve and manipulated into a total variable cost curve, and add an amount to reflect fixed resources and costs at the bottom of the total product curve. By once more turning the page over and holding it up to the light, you should be able to see a typical total cost (TC) function, as shown in Panel C of Figure 8-3.

Average and Marginal Costs in the Short Run In the previous chapter you learned that the slope of a ray from the origin to the total product curve equals the average physical product of labor (or APP_L), while the slope of the total product curve reflects the marginal physical product of labor (or MPP_L). Similarly, the average cost curves and the marginal cost curve can be derived directly from our three total cost curves (TFC, TVC, and TC).

Three rays are drawn from the origin of Figure 8-4 (Panel A) to the *TFC* curve. The slope (m) of each of these rays is determined by dividing *TFC* by the quantity of output, which is, by definition, average fixed costs (*TFC*/*Q*). For example, the slope of the ray m=60 is \$3,000/50 or \$60. Following the dotted lines down from the intersection of the rays with the *TFC* curve to Panel B yields the corresponding value of the *AFC* curve. Since *TFC* is constant, the *AFC* curve declines continually and forms a *rectangular hyperbola*–a curve with a constant area underneath each point on the curve.

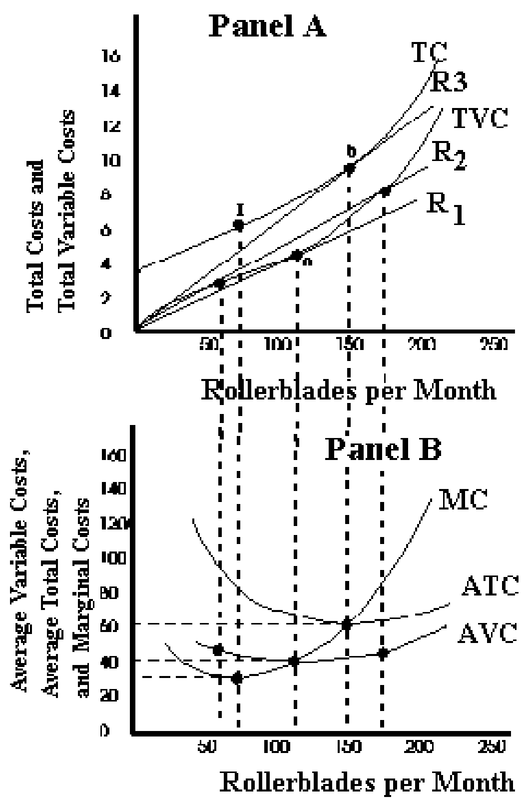
Figure 8-4 Deriving the Average Fixed Cost Curve



The slope (*m*) of a ray from the origin to the *TFC* curve reflects *AFC* (*TFC/Q*) at the point where the ray intersects the *TFC* curve. Rays *m=60*, *m=30*, and *m=15* reflect corresponding values of *AFC*, which allow derivation of the *AFC* curve in Panel B. Since *TFC* is constant, the *AFC* curve falls continuously, forming a *rectangular hyperbola*.

Average variable and average total cost curves are determined by drawing rays from the origin to the *TVC* and *TC* curves respectively. Ray R_2 in Figure 8-5 (Panel A) intersects the *TVC* curve in two places (Q = 60 and 175) and has a slope of $8,000/175 or $45.71. Following the dotted lines down to the *AVC* curve in Panel B, one sees that AVC are $45.71 when output is either 60 or 175 rollerblades. The *AVC* curve reaches its minimum when a ray (R_1) from the origin is just tangent to the TVC curve. This occurs at an output of 110 rollerblades (point *a*). As output is increased up to 110 rollerblades, *AVC* falls (the slopes of rays from the origin to the *TVC* curve fall steadily as output is increased), but beyond 110 rollerblades, *AVC* rises (the slope of rays from the origin increase along with increasing output). The same logic also applies to the *ATC* curve, which reaches its minimum point when 150 rollerblades are produced (ray R_3 is just tangent to the *TC* curve at this level of output).

Figure 8-5 Deriving the *AVC*, *ATC* and *MC* Curves



The marginal cost curve can be derived from either the *TVC* or *TC* curve because these curves are vertically parallel. More specifically, the slope of either the *TC* or *TVC* curve at any given point equals the marginal cost of producing the corresponding output. Recall that the slope of a single point on a curve equals the slope of a tangent to that point. Thus, the marginal cost of producing at point *a* (110 rollerblades) equals the slope of ray R_1 in Panel A of Figure 8-4. The slope of ray R_1 also corresponds with the minimum value of the *AVC* curve, so the *MC* curve intersects the *AVC* curve at its minimum point (see Panel B). Likewise, the slope of ray R_3 reflects marginal costs at point *b* (150 rollerblades), which corresponds to the minimum value of the *ATC* curve.

An inflection point (point *I*) occurs on the *TC* and *TVC* curves when variable costs, which have been rising at a decreasing rate, begin to rise at an increasing rate. Thus, the inflection point coincides with the minimum value of marginal cost, which occurs at an output of 70 rollerblades in Panel B of Figure 8-5.

Average variable and average total costs reflect the slopes of rays drawn from the origin to the *TVC* and *TC* curves respectively. Knowing the slope of a ray and output level where the ray intersects the *TVC* or *TC* curve lets you derive the *AVC* and *ATC* curves in Panel B. The minima of the *AVC* and *ATC* occur when a ray emanating from the origin is just tangent to the *TVC* or *TC* curves. Ray R_1 identifies the minimum value of the *AVC*, and ray R_3 identifies the minimum value of the *ATC*. The *MC* curve requires measuring the slope of the *TVC* or *TC* curve at various points. This is accomplished by calculating the slope of a line tangent to the point on the *TVC* or *TC* curves. Rays R_1 and R_3 define the two points on the marginal cost curve that respectively correspond with the minima for *AVC* and *ATC*. Marginal cost is at its minimum at the inflection point (*I*) of the *TVC* and *TC* curves, so the *MC* curve attains its minimum at the corresponding output level (70 rollerblades).

Average Physical Product and Average Variable Costs In the short run, production costs are tightly linked to the productivities of variable resources. At your rollerblade firm, the productivity of your workers (your only variable resource) influences and ultimately determines the shape of your cost curves.

Mathematically, *AVC* equals the wage rate (*w*) paid to labor (*L*) multiplied by the inverse of the APP_L:

$$AVC = TVC/Q = wL/Q = w/Q/L = w/APP_L = w(1/APP_L)$$

Marginal Physical Product and Marginal Costs Moreover, *MC* equals the wage rate multiplied by the inverse of the MPP_L:

$$MC = \Delta TVC/\Delta Q = \Delta(wL)/\Delta Q = w/\Delta Q/\Delta L = w/MPP_L = w(1/MPP_L)$$

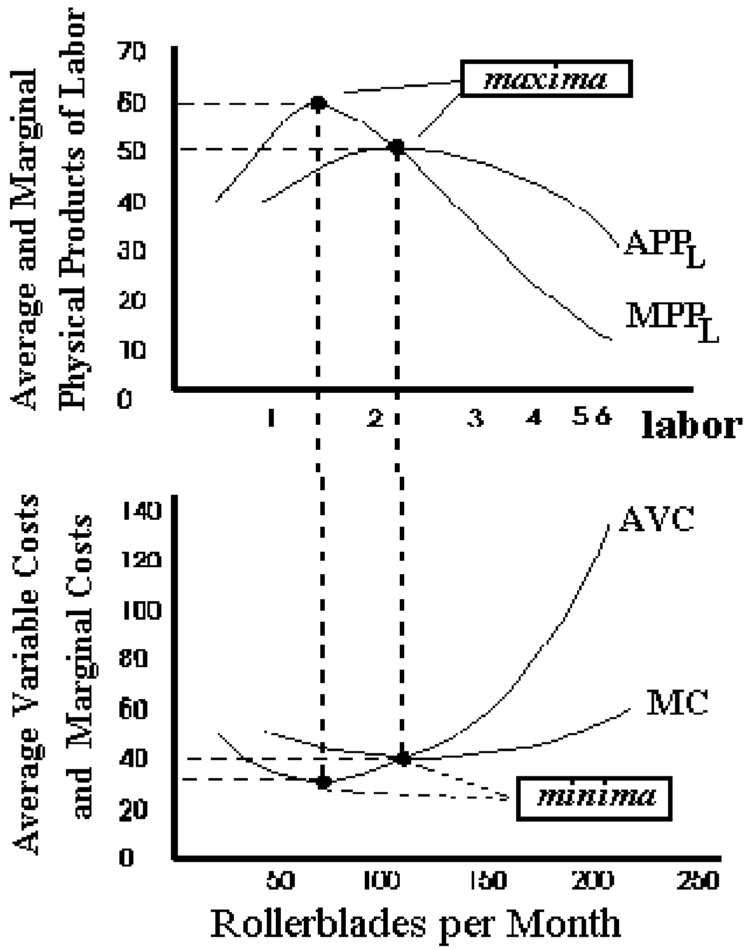
Table 8-3 reflects these equations. The APP_L for the first worker hired is 40, and *TVC* equal the monthly wage rate of $2,000. Dividing $2,000 by 40 yields $50, which equals *AVC*. When a second worker is hired, production increases (MPP_L) by 60 rollerblades. Dividing the wage rate of $2,000 by 60 yields *MC*, which equals $33.33.

Table 8-3 Relating Productivity and Costs at Radical Rollerblades

Labor (L)	Output (Q)	Average Product of Labor (APPL)	Marginal Product of Labor (MPPL)	Total Variable Costs (TVC)	Average Variable Costs (AVC)	Marginal Costs (MC)
0	0	0	0	--	--	--
1	40	40	40	$ 2000	$50.00	$50.00
2	100	50	60	$4000	$40.00	$33.33
3	145	48.33	45	$6000	$41.38	$44.44
4	175	43.75	30	$8000	$45.71	$66.67
5	195	39	20	$10000	$51.28	$100
6	210	35	15	$12,000	$57.14	$133.33

Take another look at both cost equations. Notice that *AVC* and *MC* equal the wage rate multiplied by the inverse of the APP_L ($1/APP_L$) and the MPP_L ($1/MPP_L$) respectively. What does this suggest about the relationship between the productivity curves (APP_L and MPP_L) and the cost curves (*AVC* and *MC*)? Put simply, the cost curves "mirror" the productivity curves. As Figure 8-6 shows, the *AVC* curve falls when the APP_L curve rises, rises when the APP_L curve is falling, and reaches a minimum when the APP_L curve is at a maximum. Intuitively this should make sense because when the APP_L is at a maximum, the amount spent on labor per unit of output (AVC) should be at its lowest point. The *MC* and MPP_L curves have a similar relationship — the *MC* curve is at a minimum when the MPP_L curve attains its maximum.

Figure 8-6 **Relationships Between *APP$_L$*, *MPP$_L$*, *AVC*, and *MC***



Average variable costs equal the wage rate divided by the inverse of the *APP$_L$* (*1/APP$_L$*). This inverse relationship causes the *AVC* curve to "mirror" the *APP$_L$* curve. The *AVC* curve falls when the *APP$_L$* curve is rising, rises when the *APP$_L$* falls, and obtains its minimum value when the *APP$_L$* curve reaches its maximum. A similar inverse relationship characterizes marginal cost and the *MPP$_L$*, so the *MC* curve bottoms out when the *MPP$_L$* curve peaks. (The quantity of labor in Panel A is disproportionately spaced so that the output produced by each extra worker corresponds the actual level of output in Panel B.)

To simplify the analysis of short run production and costs, only labor has been allowed to vary, but the results would be qualitatively similar if we allowed all resources but one to vary. The approach to long run production costs is slightly different because all of a firm's resources are variable.

Costs in the Long Run

As owner and chief operations officer of Radical Rollerblades, you can choose the resource combination your firm will use in the long run. Labor and capital are your only productive resources, which makes things relatively easy, but how do you determine the optimal combination? The first thing you need to know is how much labor and capital you can acquire for a given total cost.

Isocost Lines

The consumer budget lines you encountered in Chapters 3 and 4 have a parallel when we analyze production costs.

> ***An*** *isocost line* ***connects all combinations of two productive inputs that can be purchased for a given total cost.***

If your operating expenses are $10,000 a month, and capital and labor each cost $2,000 a unit per month, then isocost line I_2 in Figure 8-7 reflects the choices available (5K, 0L; 4K, 1L; ...; 0K, 5L). If you increase monthly expenditures to $16,000 with the price of capital and labor unchanged, then your new isocost line will be I_3, which is parallel and further from the origin than the original I_2. And if your monthly expenditures are again $10,000, but the per unit price of capital increases to $5,000 (labor is still $2,000 a unit), your new isocost line will be I_1, which pivots down the capital axis to reflect the higher price of capital. Isocost lines are mathematically equivalent to a consumer's budget line.

Your only productive resources are labor and capital, so your total costs (*TC*) equal the wage rate (*w*) multiplied by the number of workers hired (*L*) plus the price (*r*) of capital multiplied by the units of capital (*K*) employed, or:

$$TC = wL + rK$$

Solving this equation for *K* yields the equation for an isocost line:

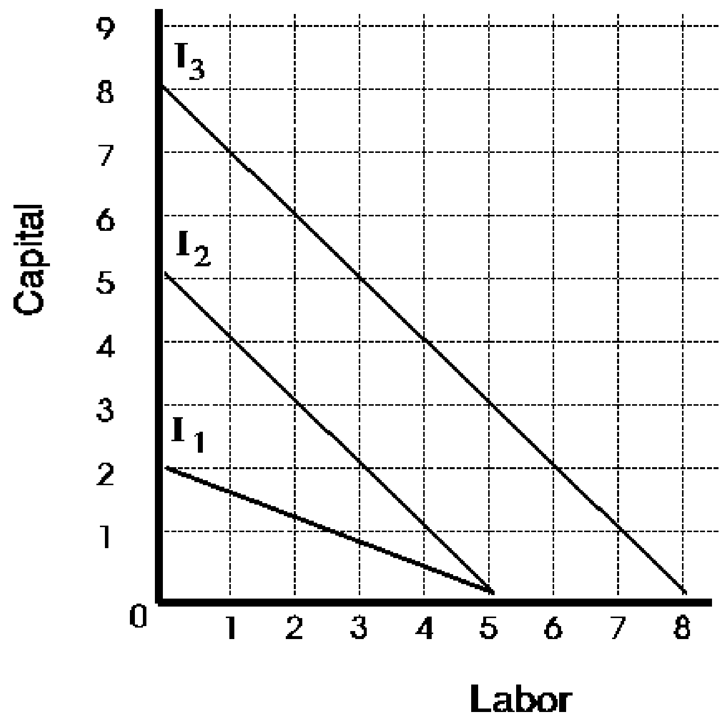
$$K = TC/r - (w/r)L$$

Total costs divided by the rental price of capital (*TC*/*r*) yields the *Y* intercept of the isocost and slope equals -*w*/*r*, which is the price of labor divided by the price of capital. We can now write the equation of isocost line I_2 in Figure 8-7, where monthly expenditures are $10,000, and capital and labor both cost $2,000 a unit:

$$K = \$10,000/\$2,000 - (\$2,000/\$2,000)L.$$

This reduces to $K = 5 - (1)L$ or $K = 5 - L$. Choosing a value for *L* (such as 3), determines the value of *K* (2, when $L = 3$).

Figure 8-7 **Isocost Lines for Radical Rollerblades**



Isocost lines show all resource combinations that can be purchased for a given cost. When labor and capital each cost $2,000 a unit per month, isocost line I_2 shows all the capital/labor combinations that could be hired for $10,000. Increasing monthly expenditures to $16,000 makes it possible to purchase all capital/labor combinations on I_3. When the price of a resource rises, the isocost line pivots down the axis of the resource that has increased in price. If monthly outlays are once again $10,000 but capital increases to $2,500 a unit, then the isocost line will "pivot" to look like I_3.

Cost Minimization

The resource combination that minimizes the cost of a given output, or alternatively, that maximizes output for a given cost, is given by the tangency of an isocost line to an isoquant. This occurs at point *b* in Figure 8-8, and is analogous to the way consumers maximize utility: You cannot produce more than 100 rollerblades with total costs of $10,000. Producing more than 100 rollerblades would require additional resources and a higher total cost (look at point *d*). For $10,000, you might have hired resource combinations shown by points *a* or *c*, but these resource combinations will produce only 80 rollerblades in a month's time. Clearly, hiring 3 units of capital and 2 units of labor (point *b*) minimizes the cost of producing 100 rollerblades, or looked at from a different viewpoint, this maximizes output when expenditures equal $10,000.

Least Cost Production

Recall that the marginal rate of technical substitution of labor for capital ($MRTS_{LK}$) at any point on an isoquant equals the slope of a line drawn tangent to that point. At the cost-minimizing resource combination (point *b* in Figure 8-7), the isocost line is tangent to the isoquant, so the $MRTS_{LK}$ equals the slope ($-w/r$) of the isocost line, or:

$$MRTS_{LK} = -w/r$$

The $MRTS_{LK}$ also equals the ratio of the marginal products of labor to capital ($-MPP_L/MPP_K$), so we can rewrite this equation as:

$$-MPP_L/MPP_K = -w/r$$

Taking the absolute value and cross multiplying yields:

$$MPP_L/w = MPP_K/r$$

This equation tells us that costs are minimized (or output is maximized) when the extra output (*MPP*) from the last dollar spent is the same for both labor and capital.[1] Suppose, for example, that the MPP_L is 4, the MPP_K is 2, and that the unit price of labor (*w*) and capital (*r*) are both $1. In this case, the last dollar spent on labor provides twice as much extra output as the last dollar spent on capital. It would be possible to produce the same output at a lower cost by reducing spending on capital by a dollar and increasing spending on labor by 50 cents. Thus, cost minimization requires the ratio of marginal physical products to resource price to be identical for all resources, regardless of the total "doses" of resources employed. This is similar to the utility-maximizing condition met when the last dollars spent on each good purchased yield equal marginal utilities ($MU_a/P_a = Mu_b/P_b$).

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[1] We can also show the cost minimizing condition by using calculus and a Lagrangian expression (*Z*) to form a constrained optimization problem — minimizing costs at a particular level of output. The appropriate Lagrangian function is:

$$Z = wL + rK + \lambda[F(L,K) - Q]$$

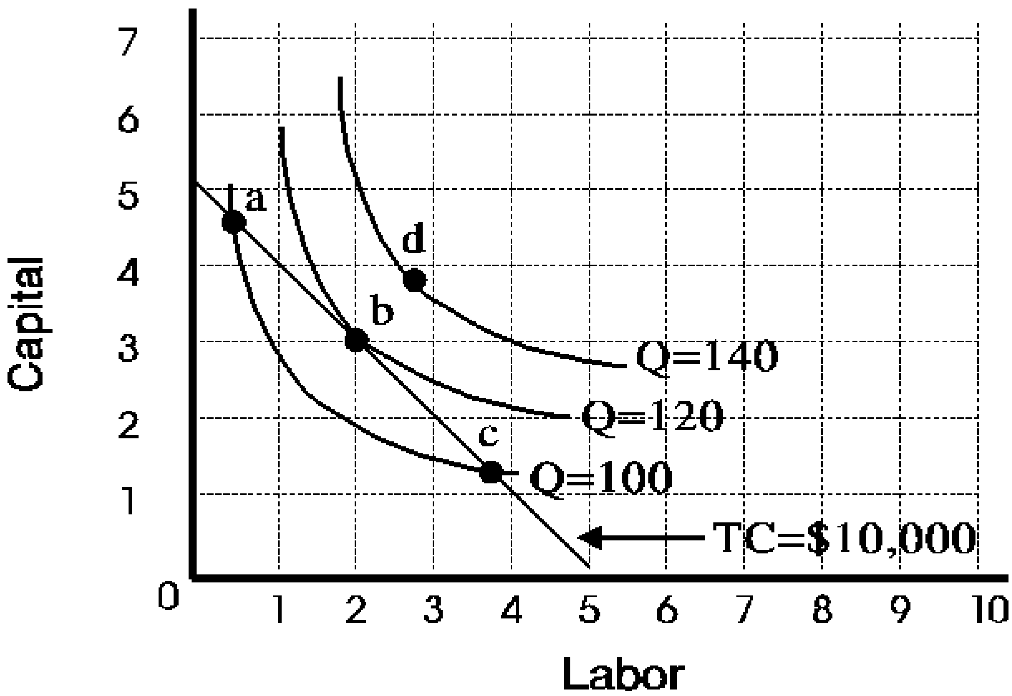
Next we take the partial derivative of *Z* with respect to *L* and *K* and set it equal to zero to find the minimum:

$$\delta Z/\delta L = w - \lambda(\delta Q/\delta L) = 0$$
$$\delta Z/\delta K = r - \lambda(\delta Q/\delta K) = 0$$

Dividing the two equations above by each other and rearranging yields the cost minimizing condition:

$$(\delta Q/\delta L)/w = (\delta Q/\delta K)/r = MPP_L/w = MPP_K/r$$

Figure 8-8 Least Cost Production at Radical Rollerblades



Tangencies between isoquants and isocost lines identify least cost production for specific output levels. At point b, 3 units of capital plus 2 units of labor is the cheapest resource combination for producing 100 rollerblades. Alternatively, the resource combination at point b shows the maximum output (100 rollerblades) that can be produced for a given cost ($10,000). At the point of least cost production, the ratio of marginal products to price is equal for both inputs (MPP/w = MPP/r), which means that the extra output from the last dollar spent is the same for both capital and labor.

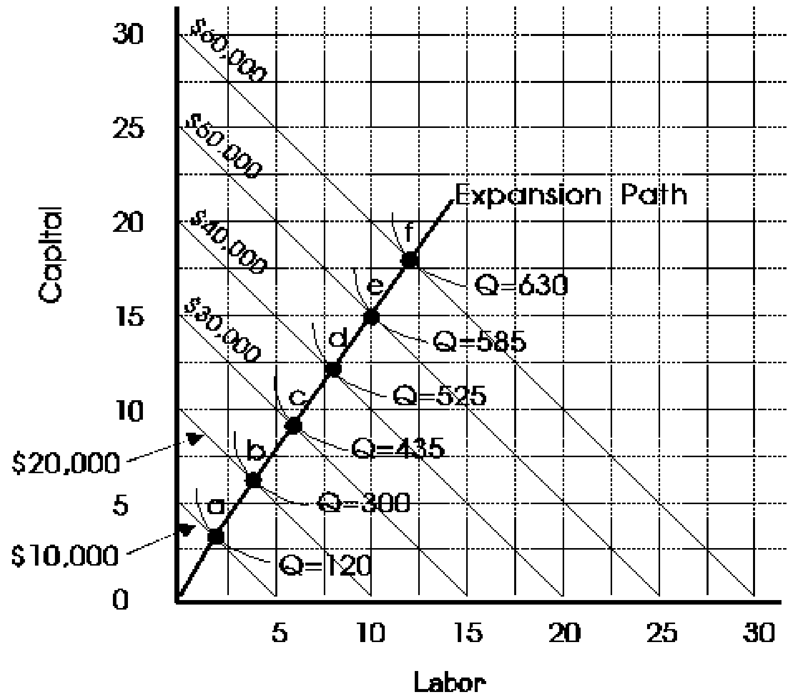
Expansion Paths and Long Run Total Cost Curves

As your firm grows, you will need to hire additional capital and labor to increase your output. Being prudent, you will want to produce your ever-expanding output at the lowest possible cost. This requires hiring a resource combination at a tangency between an isocost line and an isoquant. Combinations of capital and labor that minimize your costs at various output levels are shown in Figure 8-9. The parallel isocost lines indicate that the relative prices of capital and labor are the same as more units of each resource are hired. The line from the origin through the points of tangency is your expansion path.

> ***An*** *expansion path* ***connects the least cost resource combinations for producing each level of output, and is given by drawing a line from the origin through the tangencies between various isocost lines and isoquants.***

A firm's expansion path is analogous to the income consumption curve for a consumer discussed in Chapter 4.

Figure 8-9 Expansion Path for Radical Rollerblades

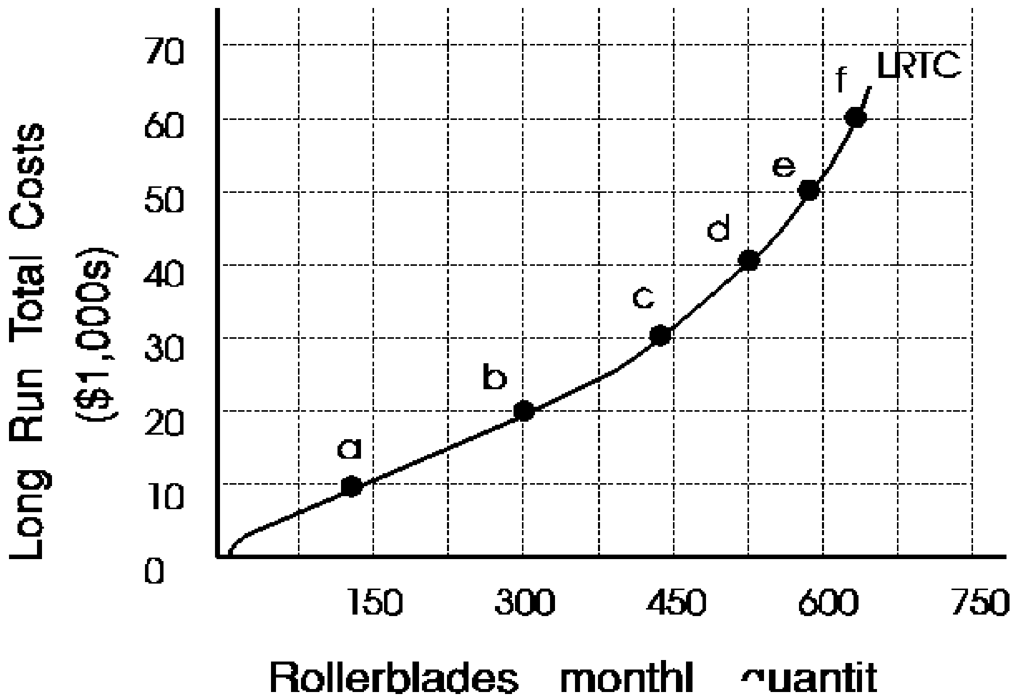


An expansion path shows least cost combinations of resources for various levels of output and is generated by drawing a line from the origin through the tangencies of isocost lines with isoquants.

An expansion path allows derivation of your long run total cost (*LRTC*) curve for Radical Rollerblades, shown in Figure 8-10. Notice that points *a-f* on your *LRTC* curve correspond to points *a-f* in Figure 8-9. Point *b* in Figure 8-9, for example, shows that the minimal cost of producing 300 rollerblades monthly is $20,000. These cost and quantity data are then used to derive point *b* in Figure 8-10. All the other points on your *LRTC* curve are generated similarly, so the *LRTC* curve shows the minimum total costs incurred in the long run as output varies.

Average Cost and Marginal Cost in the Long Run The long run average cost (*LRAC*) and long run marginal cost (*LRMC*) curves are derived in the same manner as their short run counterparts. The slope of a ray from the origin to the *LRTC* curve defines *LRAC* (*LRTC/Q*), which reach a minimum value when a ray from the origin is just tangent to the *LRTC* curve. The slope of the *LRTC* curve itself determines *LRMC* ($\Delta LRTC/\Delta Q$), which "bottoms out" at the inflection point of the *LRTC* curve. Figure 8-11 illustrates these relationships, which partially parallels the earlier derivation of short-run average and marginal cost curves.

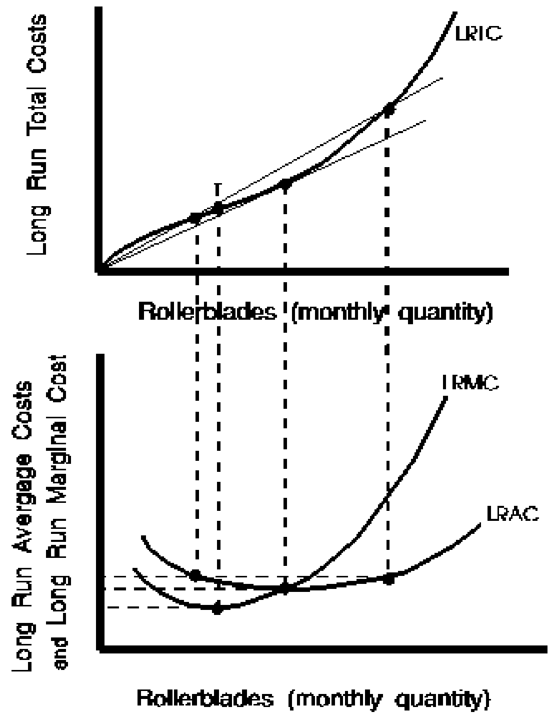
Figure 8-10 Long Run Total Cost (*LRTC*) Curve for Radical Rollerblades



The long run total cost (*LRTC*) curve is derived from the expansion path, and shows the minimum total costs incurred at various levels of output. Points *a-f* correspond with points *a-f* in Figure 8-9.

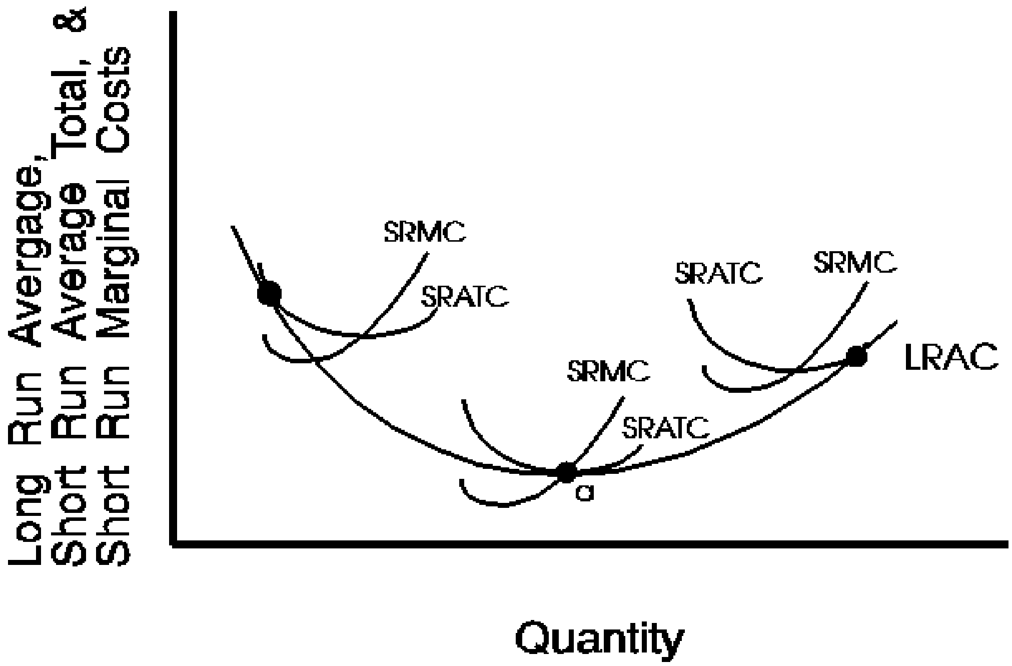
How short-run and long-run average costs are related is shown in Figure 8-12. Only three representative short run average total cost (*SRATC*) curves are shown, but a multitude of *SRATC* curves actually correspond to every possible plant size. The *LRTC* curve is just tangent to the *SRATC* curves and forms an *envelope curve* reflecting which plant sizes are associated with the minimum average costs of producing each possible level of output. Notice, however, that the *LRTC* curve is not tangent to the minimum point on each *SRATC* curve. Only at the minimum point of the *LRTC* (point *a*) is there a tangency between the minimum point on a *SRATC* curve and the *LRTC* curve.

Figure 8-11 Derivation of Long-Run Average and Marginal Cost Curves



The slope of a ray from the origin to the *LRTC* curve defines long run average costs (*LRAC*) at the output level where the ray intersects the *LRTC* curve. This information (value of slope) is then used to derive the *LRAC* curve in the bottom graph. The *LRAC* curve reaches its minimum value when a ray from the origin is just tangent to the *LRTC* curve. The slope of the *LRTC* curve itself provides long run marginal cost (*LRMC*) which is shown in the bottom graph. The *LRMC* reaches its minimum at the level of output that corresponds with the inflection point (point *I*) on the *LRTC* curve.

Figure 8-12 The Long Run Average Cost Curve



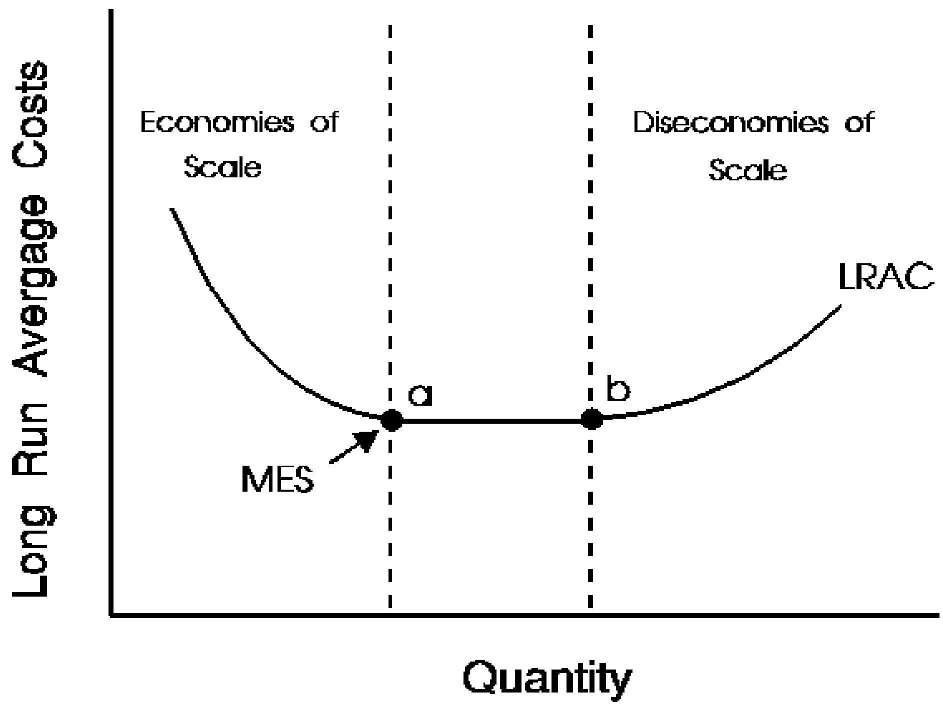
The long run average cost (*LRAC*) curve forms an "envelope" just tangent below all the short run average total cost (*SRATC*) curves. Since each *SRATC* is associated with a different plant size, the *LRAC* shows the minimum average costs of producing each level of output.

Returns to Scale and the Long Run Average Cost Curve

Recall that the proportional growth in output due to an increase of productive inputs is known as returns to scale. The shape of the *LRAC* curve is determined by the returns to scale a firm experiences as it grows. Increasing returns to scale (*IRS*) result in a negatively-sloped LRAC curve, constant returns to scale (*CRS*) yield a horizontal *LRAC* curve, and decreasing returns to scale (*DRS*) generate positively-sloped *LRAC* curves. This should make intuitive sense because output which grows proportionally larger than inputs (*IRS*) should result in reduced costs per unit of output (average costs), while output which grows proportionally less than inputs (*DRS*) should yield increasing costs per unit of output. When output grows in proportion to inputs (*CRS*), costs per unit of output should be unchanged. Most firms experience the whole gamut of returns to scale (first *IRS*, then *CRS*, and finally *DRS*), which explains the ***U***-shape of the *LRAC* curve.

When *LRAC* are falling, *economies of scale* are said to exist, while rising *LRAC* portend *diseconomies of scale*. Up to point *a* in Figure 8-13, economies of scale occur, while diseconomies of scale are experienced beyond point *b*. Between points *a* and *b*, neither economies nor diseconomies of scale occur because *LRAC* are constant.

Figure 8-13 Returns to Scale and the Long Run Average Cost Curve



Returns to scale explain the shape of the *LRAC* curve. The negatively-sloped, flat, and positively-sloped portions of this flattened U-shaped *LRAC* curve are attributable to increasing, constant, and decreasing returns to scale. Economies of scale exist when *LRAC* falls with increased output, while diseconomies of scale occur when *LRAC* rises as output is increased. Point *a* identifies minimum efficient scale (*MES*) — the smallest plant size that will produce at minimum *LRAC*.

The ranges where economies or diseconomies of scale are actually encountered vary substantially among industries. Engineering estimates and the few statistical studies of cost functions that are available indicate that there are typically substantial ranges of output for which average costs are roughly constant, as depicted in the middle of the *LRAC* curve in Figure 8-13.

An idea known as the *survival principle* suggests that clustering within an industry of firms or plants of a particular size is conclusive evidence about efficient scales of operation. Some economists have tried to apply this principle to specific industries. Critics, however, argue that survival depends on a multitude of factors (luck, monopoly power, business acumen, growth or decline of an industry, and so on) and thus, that some inefficient firms may survive, while some efficient firms fail.

> *Minimum efficient scale (MES)* ***plants are the smallest that will produce output at minimum average total cost.***

Minimum efficient scale (point *a* at the beginning of the flat portion in Figure 8-13) has been estimated for various industries using accounting data, engineering estimates, and the survival technique. Typically, *MES* is reported as a percent of the total market.

Figure 8-14 presents some estimates of *MES* for selected industries here and abroad. Measuring long-run costs is unavoidably imprecise, but the concept is still useful in analyzing industry adjustments to changes in demands, resource prices, or other events.

Figure 8-14 Minimum Efficient Scale for Selected Industries

Technological Change and Costs

We have treated technology as constant and immutable to this point in our analysis. We know, however, that technological advance presents firms with opportunities to increase their productivity and reduce production costs. By some estimates, technological change accounts for over 20 percent of long-term economic growth and over half of our productivity gains during the last half century.[2]

Profit opportunities may induce new technologies that change outputs and production techniques or improve resources, and ultimately drive down costs. For example, the advent and widespread acceptance of the personal computer (PC) gave rise to a multitude of companies that produced software which catering to every imaginable business need. Today it is almost impossible to find firms that do not rely on PCs and powerful software to conduct business. In the pursuit of profits, the software companies changed the way most firms do business.

Technological Advance as a Response to Profit Incentives

Technological advances have often resulted from tinkering, undirected research, or serendipity. The list of goods which fall into this category is almost endless: telephones, automobiles, planes, radios, TVs, plastics, computers, lasers, gortex, etc.. Major technological breakthroughs tend to arrive in waves that influence numerous industries or forms of production. Predicting the direction and timing of such sweeping technological advances is impossible, so technological change in many cases can be viewed as an external or exogenous force upon the production process.

[2] Edward F. Dennison, "Contributions to 1929-82 Growth Rates," *Trends in American Economic Growth*, 1929-82, the Brookings Institute, 1985.

International Markets as Forces for Change

International trade also facilitates technological advance. Goods, services, and processes which embody the latest technology are exported and imported throughout the world. This diffusion of technology permits firms to incorporate resources and ideas which boost their productivity worldwide, lowering their costs. As international markets become more sophisticated, potential profits from innovative technologies grow, further spurring efforts on research and development. Worldwide diffusion of technology through international trade also increases the chances of technological advance or spinoff technologies, because more minds are exposed and stimulated by the original technological advance.

It is uncertain whether giant or small enterprises are systematically favored by technological advance. Some new technologies enhance economies of scale; others work best when an operation is small. We can be sure, however, that technological advances make options available that reduce average production costs.

Exogenous Technological Change

Chapter Review

1. Production costs can be divided into *fixed* and *variable costs*. Rent payments for building space and loan payments for capital are examples of the fixed costs incurred even if zero output is produced. Wages paid to workers, utility bills, and payments for raw materials are examples of variable costs, which depend on the level of production.

2. *Total costs (TC)* include total fixed costs (*TFC*) and total variable costs (*TVC*). *Total fixed costs* are unrelated to the level of output produced, and are incurred for resources that are fixed (usually capital) in the short run. *Total variable costs* are incurred when variable resources (usually labor) are hired, and are related to the level of output, increasing as output rises. *TC* and *TVC* initially rise at a decreasing rate, but begin to rise at ever-increasing rates as diminishing marginal returns set in.

3. *Average total costs (ATC)* are total costs per unit of output and can be determined by summing average fixed costs (*AFC*) and average variable costs (*AVC*). Dividing total fixed costs by output yields *average fixed costs*, which continually decline as output is increased. *Average variable costs* are total variable costs per unit of output, and usually generate a U-shaped curve when graphed. The U shape of *ATC* and *AVC* curves is explained by diminishing marginal returns.

4. *Marginal cost* is the change in total cost required to produce an additional unit of output, and can be calculated by dividing the change in *TC* or *TVC* by the change in output. The marginal cost curve is U-shaped because of diminishing marginal returns, and intersects the *AVC* and *ATC* curves at their minimum points.

5. The *AFC* curve is obtained by drawing a ray from the origin to the *TFC* curve. The slope of the ray equals *AFC* at the level of output where the ray intersects the TFC curve. Similarly, the *AVC* and *ATC* curves are reflected by the slopes of rays emanating from the origin which intersect the *TVC* and *TC* curves respectively. The marginal cost curve can be calculated from the slope of the *TVC* or *TC* curve.

6. Average variable costs (*AVC*) and marginal cost (*MC*) are inversely linked to the average product of labor (APP_L) and the marginal product of labor (MPP_L) respectively. Both cost curves (*AVC* and *MC*) reach their minima when the corresponding productivity curves (APP_L and MPP_L) reach their maxima, decline when the corresponding productivity curves are rising, and rise when their corresponding productivity curves are falling.

7. *Isocost lines* show all input combinations that can be purchased for a given cost. Algebraically, isocost lines can be expressed as $K = TC/r - (w/r)L$, where TC/r gives the *Y* intercept and $-(w/r)$ is the slope.

8. Least cost production occurs where an isocost line is just tangent to an isoquant. At that tangency, the ratios of marginal products to price are equal for both inputs ($MPP/w = MPP/r$), which means that the extra output from the last dollar spent is identical for both capital and labor.

9. An *expansion path* connects the least-cost resource combinations, for all possible output levels, and is generated by drawing a line from the origin through the tangencies of isocost lines with isoquants.

10. The *long run total cost (LRTC) curve* is derived from a firm's expansion path, and shows the minimum total costs that a firm incurs at various levels of output.

11. *Long run average costs (LRAC)* equal *LRTC* divided by output (*LRTC/Q*), and are derived graphically by determining the slope of a ray drawn from the origin to the LRTC curve. *Long run marginal costs (LRMC)* equal the slope of the *LRTC* curve ($\Delta LRTC/\Delta Q$), and are determined graphically by drawing lines just tangent to various points on the *LRTC* curve.

12. The long run average cost (*LRAC*) curve forms an "envelope" which is just tangent below the short run average total cost (*SRATC*) curves. Since each *SRATC* is associated with a different plant size, the *LRAC* shows the minimum average costs of producing each possible level of output.

13. Returns to scale explain the shape of the *LRAC* curve. The negatively sloped, flat, and positively sloped portions of a typically U-shaped *LRAC* curve are attributable to increasing, constant, and decreasing returns to scale respectively. *Economies of scale* exist when *LRAC* fall with increased output, while *diseconomies of scale* occur when *LRAC* rise as output is increased.

14. *Minimum efficient scale (MES)* is the smallest plant size that will allow a firm to produce at minimum *LRAC*.

Exercise 8-1 Indifference analysis and isoquant/isocost analyses have numerous parallels.

1. Consumer budget lines and isocost lines are similar.
2. Typical indifference curves and isoquants are both convex to the origin.
3. Consumer equilibrium require tangency between the budget line and the highest possible indifference curve, while least cost production requires tangency between an isocost curve and an isoquant.

What are major analytical differences between these types of analysis in terms of:

1. the objectives of consumers versus those of firms?
2. measurements on indifference curves versus those for isoquants?

(Hint: Does tangency between an isocost and an isoquant ensure producer equilibrium? Why not? Does tangency between the consumer's budget line and an indifference curve ensure consumer equilibrium? Why?)